Finding rules and describing patterns — Year 1/Year 2

Teddy's birthday candles

Teddy is 7 years old today.

How many birthday candles has he blown out since he was born?

Resources

- Teddy bear and birthday cake with 7 candles (or use clip art on an interactive whiteboard)
- Drinking straws cut into 2-inch lengths

Objectives

- Describe and extend simple number sequences
- Recognise simple patterns and relationships, generalise and predict
- Solve and explain their solution to a given problem

By the end of the lesson, children will be able to:

Vocabulary

pattern sequence continue predict

Necessary prior knowledge

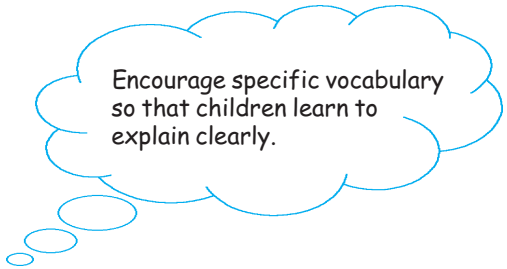
Counting, knowledge of the number system
Simple addition
More than, less than
Recognise, say, copy and extend simple patterns

  Ref: DfES 1387-2005 G

Main teaching activity

Set the scene for the problem by saying that today is Teddy's birthday.
Show Teddy with his birthday cake with seven candles on it.

Q. How old is Teddy today?
Q. How do you know?
Q. How many candles would he have had on his cake last year?
Q. How many will he have next year?
Q. How many would he have had two years ago?
Q. How do you know?



If children are unsure, add and remove candles to show the number for each question.

Ensure that children have used language such as one more, one less, same number as how old he is, etc. Ask similar questions, ensuring that all children are clear.

Light the seven candles on the cake and sing 'Happy Birthday' to Teddy and help him blow the candles out.

Say that Teddy has just blown out seven candles.
But he didn't blow out that many when he was only one.

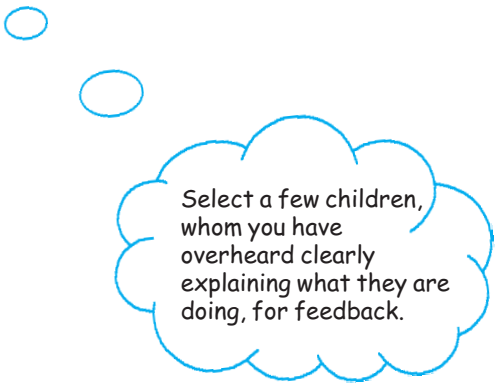
If you are using clip art on an interactive whiteboard, copy and paste the first cake and then add one more candle.

Q. How many candles did Teddy blow out when he was one?
Q. How many candles had he blown out by the time he was two?

Show this on two cakes.
Make sure the children understand that Teddy has blown out this year's two candles but also last year's one candle and that they need to be added together.

Discuss how you might record this in a way that is clear to everyone. Suggestions might include using cut-up drinking straws as candles, pictures, tallies, or calculations (1, 1 + 2, …), or a mixture of these.

Ask children to work in pairs, choose a method of recording and use this to show the number of candles for Teddy's 1st, 2nd, 3rd and 4th birthdays.



Drawing together

Choose a variety of recordings, but no more than three, e.g. a picture recording, a straw recording and a calculation recording, and invite children to explain what they have done.

Q. Which way of recording is clear and helps us to work out what we need to know quickly?

Q. Which helps us to see how the pattern of the candles grows?

Q. Could we find out how many candles Teddy will have blown out next year, from these recordings?

Ensure that children are aware of:

- the cumulative nature of the number of candles;
- the need to record in clear columns to see the pattern growing.

Show children how to move from using straws to using pencil and paper, keeping recording in columns, e.g.

I	1
I II	3
I II III	6

Differentiation may be necessary for those children who are not secure with counting, or totalling to 28, and for those who find the answer quickly. You may ask some children to find the number of candles blown out when Teddy is 5, or 8.

Now ask the children:

Q. If Teddy is 7 years old today, what is the total number of birthday candles he has blown out since he was born?

Explain that they won't be able to answer the question straight away. Ask children to check that their partner understands what the question is asking and to discuss what they will do to solve it. Say that they may change the way they record if they can now see a better way of doing it, or they may stay with their own recording.

This paired discussion gives an opportunity for children to develop their reasoning skills and use of vocabulary.

Allow children time to work on the problem in pairs.

Circulate and observe surprises – children who exceed expectations or don't meet expectations. Be aware of a dominant partner, and target questions at the passive partner to ensure understanding.

As you work with one or more groups, ask children to clearly show their recording for the 4th or 5th birthday and to talk about it using specific language. These may be given on cards, if the child is able to read them, e.g. more, less, altogether, as well as.

If the language used is not concise ask them to use the specific language and to practise their explanation with someone else.

When they can do this have some prepared questions ready, e.g.

Q. What if Teddy was 8? Could you solve this without writing or drawing?

Q. What language would you use to explain it? What would be the important words?

Q. What questions can you ask a partner?

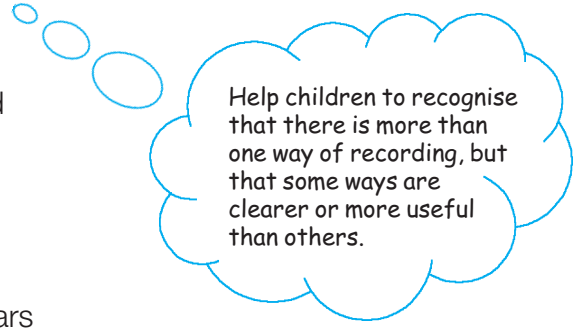
When most children have solved the problem ask them to share their thinking with another pair at their table so that all four understand each other's work.

Practising their explanations with a partner can build confidence and provide you with some exemplar pairs to draw upon in the plenary.

Plenary

Collect answers to the 'seven candles' question with the class. If there are different answers, explore why and deal with misconceptions.

Q. Was your recording easy to understand?
Q. Did anyone change their way of recording because they found a better way?
Q. Can you explain what was clearer about it?
Look for examples of other clear recordings that show the accumulated facts and a clear column structure.



Discuss patterns that children have noticed.
Draw out:

- increasing patterns from recording, i.e. that we include all the years before and then this year's birthday;
- number patterns, e.g. in recording
  1 + 2
  1 + 2 + 3

Other patterns which children may have noticed are:

- In the solution 1, 3, 6, 10, 15, 21, 28
  the differences are 2, 3, 4, 5, 6, 7
  i.e. the last number that was added is the new age of Teddy.

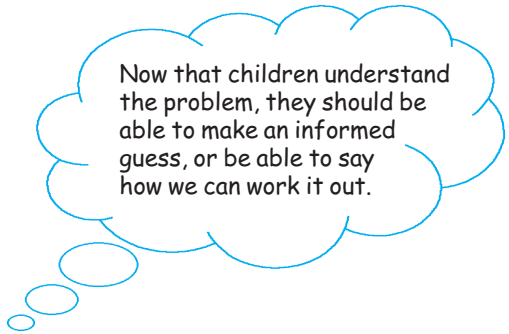
  Ref: DfES 1387-2005 G

- Triangle shapes are made by the way in which the candles are arranged, represented by straws or lines.

```
I
I  II
I  II  III
I  II  III  IIII
I  II  III  IIII  IIIII
I  II  III  IIII  IIIII  IIIIII
I  II  III  IIII  IIIII  IIIIII  IIIIIII
```

Review how the following recording was easier for some of us to add up, and make reference to bonds to 10 if some children noticed this.

```
1
1  2
1  2  3
1  2  3  4
1  2  3  4  5
1  2  3  4  5  6
1  2  3  4  5  6  7
```



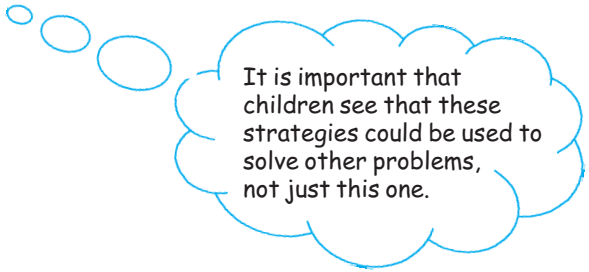
Q. If Teddy was 8, can you predict how many candles he would have blown out? For those who have done this, ask how many candles Teddy would have blown out if he was 10.

Q. How could we test this?
Invite one child to do this.

Q. What were the most useful strategies we used to help us answer the problem?

Q. Can we make a list to help us remember for next time, e.g.

- talking about the question to understand what it is asking us;
- clear recording;
- talking and explaining;
- looking for patterns.



Finding rules and describing patterns — Year 3/Year 4

Birthday candles

Sally Gillespie is 10 years old today.

How many birthday candles has she blown out on her birthday since she was born?

Objective

- Solve mathematical problems or puzzles, recognise and explain patterns and relationships, generalise and predict. Suggest extensions by asking 'What if . . . ?'

By the end of the lesson, children will be able to:

- recognise, describe and continue a number sequence;
- explain their generalisations.

Vocabulary

pattern sequence continue predict rule

Necessary prior knowledge

Counting, knowledge of the number system
Knowing strategies for adding several numbers
Familiarity with more than, less than
Recognise, say, copy and extend simple patterns

Resources

- Birthday cake with 10 candles (or use clip art on an interactive whiteboard)

Main teaching activity

Having a cake with 10 candles burning and blowing them out will help to capture children's interest.

Introduce the problem by saying that Sally is 10 years old today. Explain to the children that after everyone had sung 'Happy Birthday' Sally was asked how many candles she had blown out altogether on her birthdays.

Circulate and listen to children's responses as they begin to consider how they might solve the problem.

Ask the children to discuss briefly in pairs how they think they would work out the answer to the question Sally was asked.

Take some brief feedback and then explain that Sally began to solve the problem like this.

Birthday	*Candles*	*Total number of candles*
1st	I	1
2nd	I II	3
3rd	I II III	6

Explaining to a partner allows children to clarify their thinking.

Q. Can you explain Sally's method to your partner?

Some children may need to explore the problem using simple drawings, e.g. sketching cakes and candles.

Ask the children to work with their partner to continue Sally's pattern for her 4th and 5th birthdays.

Drawing together

Q. How many candles had Sally blown out by her 4th birthday?
Her 5th birthday?

Take answers from the children. Agree that Sally has blown out 10 candles by the time she is 4 and 15 candles by the time she is 5.

  Ref: DfES 1387-2005 G

Add the 4th and 5th birthday information to Sally's pattern.

Birthday	*Candles*	*Total number of candles*
1st	I	1
2nd	I II	3
3rd	I II III	6
4th	I II III IIII	10
5th	I II III IIII IIIII	15

Inviting pairs to discuss together provides a good speaking and listening opportunity. It also helps to develop their reasoning skills.

Q. What patterns do you notice in the numbers in the total column?
Ask the children to discuss this in pairs.
Invite two pairs to join together and share the patterns they have noticed. Draw out that the number in the total column increases by a number one more than previously, the age of Sally at the time. This can also be clearly seen in the tallies of candles.

Q. Now that we understand the problem, can we predict how many candles Sally will have blown out by her 10th birthday? Do we have to find out the number blown out by her 6th, 7th, 8th and 9th birthdays?

Circulate and listen to children's predictions and reasoning.

Ask the children to work in pairs for a few moments to discuss this.

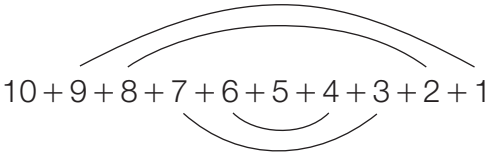
Take feedback, asking children to share their predictions and explain their reasoning. Record the predictions on the board.

Drawing together

Agree that if we add 10, 9, 8, 7, 6, 5, 4, 3, 2 and 1 we will know how many candles Sally has blown out by the time she is 10.

Q. Is there a quick way to add these numbers?
Give the children a few moments to discuss this.

Agree that looking for pairs that make 10 is an efficient method.



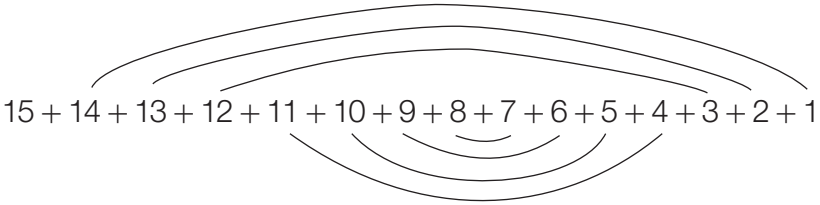
This provides a useful opportunity to remind children to identify and use efficient methods of calculation.

Model recording to show that four pairs make 10, add 5, and then add 10, making a total of 55. Make sure the children understand that this means Sally has blown out 55 candles and therefore the problem has been solved.

Q. What if we were working out how many candles Sally had blown out on her 15th birthday? What numbers would we need to add?

Q. Would we still look for pairs that make 10? What would pairs of numbers need to total?

Establish that we could look for pairs that make 15.



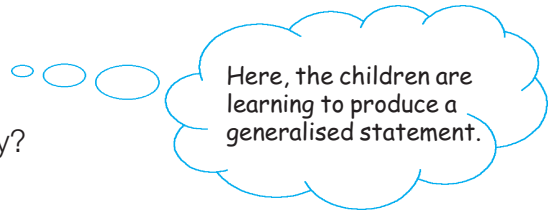
Children can investigate the number of candles Sally will have blown out at an age appropriate to their level of ability. They could choose ages, thus self-differentiating, and ask 'What if…?' questions.

Alternatively, the children might spot pairs to 20 or 10 which they can quickly add together.

Ask children to work out how many candles Sally will have blown out at different ages.

Plenary

Q. What did we do to work out how many candles Sally had blown out?
Q. Would this method always work?
Q. Using what we have learned today can you create a rule to find the number of candles someone of any age had blown out on their birthday?



Tell children that their rule should begin with this phrase:

'To find the total number of candles someone has blown out on their birthday you …'

and include some or all of these words:

'total, add, altogether, age, previous'.

Ask children to work in pairs to agree a rule.

Invite several children to share their rules and the rest of the class to evaluate the clarity of the rules shared.

  Ref: DfES 1387-2005 G

Finding rules and describing patterns — Year 5/Year 6

The candle problem

Mrs Gillespie is 73 today. She has had a birthday cake and candles every year since she was born.

How many candles has she blown out in her lifetime?

Objective

- Solve mathematical problems or puzzles, recognise and explain patterns and relationships, generalise and predict. Suggest extensions by asking 'What if . . . ?'

By the end of the lesson, children will be able to:

- express the general term in words and begin to do this algebraically.

Vocabulary

pattern sequence term predict generalise

Necessary prior knowledge

Recognise and extend number sequences
Some experience of relating a term to its position in the sequence
Know the relationship between the areas of right-angled isosceles triangles and squares

Resources

- A3 paper for paper mat activity
- Squared paper available, if required

Main teaching activity

Set the context of the problem. Mrs Gillespie is 73 today. She has had a birthday cake every year since she was born. How many candles has she blown out in her lifetime?

Ask children to work with a partner and discuss what their understanding of the problem is and what kind of recording may be useful. Remind children that recording needs to be systematic and easy to refer to.

Circulate and listen for discussion demonstrating efficient ways of recording.

Drawing together

Establish that everyone understands the problem and that everyone is clear about the method of recording they will use.

If there are several children who do not know how to record, ask a child whom you have overheard to explain their method.

Work with the uncertain children to begin with until they are secure in recording.

Allow children to explore and enjoy the problem for a short while. It is likely that the children will see that they could add $1 + 2 + 3 + \ldots + 72 + 73$, which will take a long time to calculate.

When children begin to realise that this will take a very long time, draw them together again.

Drawing together

Q. What problems have you come up against?
Discuss problems children have experienced. Pick up on the fact that finding a solution could take a very long time.

Q. What can we do so that we don't have to write down all the numbers of candles and add them up?

Ask children to take a few minutes to discuss possibilities and establish that we need to develop a rule that will get us to the answer more quickly.

  Ref: DfES 1387-2005 G

Say that you are going to go back to an arrangement of the candles to see what patterns and information can be gained from it. Say that simplifying a problem by using smaller numbers can often help us to understand how we can solve it, perhaps seeing patterns that we don't see when using larger numbers.

Suggest that the candles could be drawn like this:

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|
|  |
|  |  |
|  |  |  |
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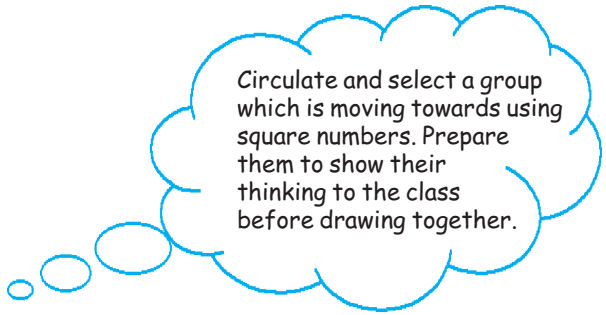
This represents the candles blown out in four years.



Q. Can you see patterns in:

- the way the diagram grows?
- the shape it makes on the paper?
- the numbers generated?
- the number of rows and columns?

Say that these may be things children want to consider.

You could use a 'place mat' activity to generate discussion and creativity in looking for useful patterns/relationships.

Put the first 4 rows in the centre of the mat. Divide the mat into 4.

Each child has a few minutes to jot down anything they notice in their space. They then share with a partner and refine their ideas, discarding anything that will not be useful. Each pair then shares with the other pair until they come up with a group consensus.

Drawing together

Ask the chosen children to describe what they see. Draw out:

- the number of rows and the number of columns are the same;
- this number is Mrs Gillespie's age, i.e. the number of candles on the most recent cake, the number of candles added to the previous total;
- the shape formed by the candles is a right-angled isosceles triangle (it looks like half of a square).

Say that when Mrs Gillespie was 5 she would have blown out the following number of candles in her lifetime:

```
|  .  .  .  .
|  |  .  .  .
|  |  |  .  .
|  |  |  |  .
|  |  |  |  |
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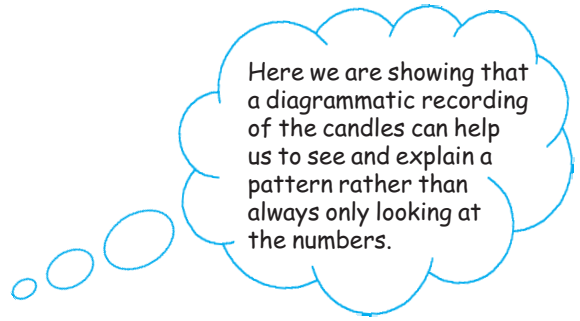


Tell the children that numbers in a sequence like this are called triangular numbers, for an obvious reason.
Check that the children understand.

Discuss the arrangement of candles and the square formed by the triangle and empty spaces.

Q. What can you say about the number of candles in the triangle, and the number if the whole square was filled?

Agree that the number of candles is a bit more than half the area of the square.

Q. Will this be the case for the 5th birthday? The 6th? The 10th? The 100th? Always?

Q. How many candles roughly would Mrs Gillespie have blown out by the time she was 6?

Discuss that this would be a bit more than half of 6×6, i.e. a bit more than 18.

Q. How many more than 18?

Encourage children to draw the diagram.
Agree that the answer is 3 more than 18, i.e. 21.

Ask the children to discuss how we might work out what to add to half of the square number, i.e. $\frac{1}{2}(n \times n)$ where n = Mrs Gillespie's age.

  Ref: DfES 1387-2005 G

Drawing together

Children may see and explain this in different ways. Ask at least one group to explain their thinking. For example, some may see that halving the square halves the number in the diagonal, but we want the whole diagonal. Draw out that the bit we need to add on to half the square is half the diagonal.

Describing what they see can help children towards a generalisation.

Q. How can we work out what half the diagonal is?
Refer back to the diagram for the 4th birthday.

```
I  .  .  .
I  I  .  .
I  I  I  .
I  I  I  I
```

Agree that in this case it is 2.

Q. What is the number in the whole diagonal?

Agree that the number in the whole diagonal is the number of candles on the most recent cake, i.e. n.

Circulate while they do this and collect children's ideas. It will be helpful if they rehearse their explanations with their group/partner/you before sharing with the class.

Q. What is the number of candles?
Agree that this is $\frac{1}{2}(n \times n) + \frac{1}{2}n$.
This can also be written as $\frac{1}{2}(n^2 + n)$.
Q. What should we do now?

Agree that it would be sensible to check that this works for the next few birthdays. Ask the children to work in pairs to do this.
Q. Are you convinced that this will always work?

This check helps children to understand and so be convinced.

Alternatively, children may see the pattern in a different way. Some may see and use the lack of symmetry in the square. Some may say that an empty row should be added to the top, as shown below.

Half of this new shape gives the correct number of candles.

Q. If n is the number of candles on the most recent cake, how can we think of the total number?

Agree that we could add n to $(n \times n)$ and then halve this total:
$\frac{1}{2}(n^2 + n)$.
[This could also be described as $\frac{1}{2} \times n \times (n + 1) = \frac{1}{2}(n^2 + n)$.]
Ask children to check this for several terms.

If you have shown two ways of looking at this, ensure that children realise that different ways of looking at the pattern have resulted in the same expression.

Ask children to solve the original problem, using a calculator if they wish.

Plenary

Ask children to share the answer:

$\frac{1}{2}(73^2 + 73)$
$= \frac{1}{2} \times 5402$
$= 2701.$

Ensure that children realise that this expression [$\frac{1}{2}(n^2 + n)$] will work for all birthdays.

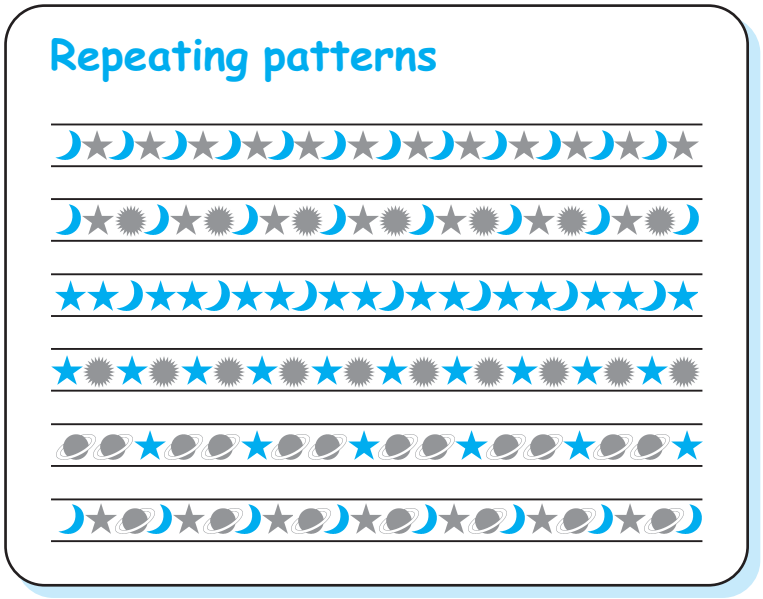
Q. If Mrs Gillespie lived to be 100, how could we work out how many candles she would have blown out?

Help the children to reflect on how we arrived at a generalisation. You may wish to draw out some or all of the following:

- being sure what the problem was;
- using diagrams to show what was happening;
- predicting and testing;
- explaining what we saw;
- recording this using mathematical language.

  Ref: DfES 1387-2005 G

Finding rules and describing patterns Foundation Stage/Year 1



Necessary prior knowledge

Some previous experience of pattern and shape would be helpful
Ordinal language

Resources

- Wallpaper borders with repeating patterns
- Gummed shapes, stamps or photocopied shapes
- Drum or tambourine
- Whiteboards
- Interlocking plastic cubes, bricks, or construction equipment

Early Learning Goals from *Curriculum Guidance for the Foundation Stage*

- Use developing mathematical ideas and methods to solve practical problems
- Talk about, recognise and recreate simple patterns

Objectives from *NNS Framework for teaching mathematics from Reception to Year 6*

Reception

- Use developing mathematical ideas and methods to solve practical problems
- Solve simple problems or puzzles in a practical context, and respond to 'What could we try next?'
- Talk about, recognise and recreate simple patterns

Year 1

- Solve simple mathematical problems or puzzles; recognise and predict from simple patterns and relationships. Suggest extensions by asking 'What could I try next?'
- Use one or more shapes to make, describe and continue repeating patterns
- Make and describe models, patterns and pictures using construction kits, everyday materials, Plasticine

Vocabulary

pattern repeat same different predict

  Ref: DfES 1387-2005 G

Activity 1

It will be helpful if the children have resources (stamps, photocopied images, stickers) which they can use later in the activity.

Ask children if they know what a pattern is.

Q. What kinds of patterns are there? Can you see any patterns in the classroom, on the walls or on the floors, around the school? Are there any patterns at home, in the wallpaper, or on your clothes?

It is important to build on children's awareness of pattern in their surroundings. Encourage them to use shape and positional vocabulary to describe them.

Show the children a pattern of objects along a wallpaper border (moon, star, moon, star, etc.)

Allow children enough time to think and talk about their own ideas before discussing with the whole class.

Q. Describe this pattern to each other.

If you have an interactive whiteboard, you could use clip art of moons and stars to copy and paste to continue the pattern.

Q. How many different objects are there? Is there a pattern? What is the pattern? Which object would come next in the pattern?

Discuss which object is first, second, third, etc.

Q. Which object will be after that? How can we check that we are right?

Introduce a second pattern with three objects, e.g. moon, star, sun, moon, star, sun, etc.

Q. How many objects are in this pattern? Which is first? Second? Third? What happens next?

Q. What will the next object be? And the next? Why?

  Ref: DfES 1387-2005 G

Show the children a pattern using two objects where one is repeated, e.g. star, star, moon, star, star, moon, etc.

Q. How many objects are in this pattern? What is first? Second? Third? And then what happens?

Q. What object will be next? And the next?

Give the children the resources (e.g. stamps, stickers, photocopied shapes) to make their own patterns using the same objects. Ask them to make different patterns to yours. They should discuss what is the same and what is different about their patterns.

Clarify children's understanding of the terms 'same' and 'different/difference'. Encourage the use of them when children are in discussion pairs. Ensure that they see that there are many different possibilities.

Encourage children to use mathematical vocabulary about shape and position.

Drawing together

Ask children to close their eyes while you cut a border into three pieces.
Q. How can we reorder these to create a pattern? What clues can we use?
Ask children to hide part of their pattern while their partners close their eyes.
Q. What's hidden? How do you know?

You could also use a 'slidy box' format or hide-and-reveal facility on an interactive whiteboard.

Activity 2

Think about referring to pattern on an interactive display to support some children when they are discussing pattern generally.

Ask the children to sit in a circle.

Explain that you are going to show children a pattern, and that children need to describe what the pattern is. Ask the first child in the circle to stay sitting, the second child to stand up, the third child to stay sitting, the fourth child to stand up, and so on.
Stop at a given point and ask:
Q. What am I going to ask the next child to do? Explain how you know.

Initial assessment opportunities are important here through focused and targeted questioning.

The children could also be clapping, clicking fingers, doing arm movements, crossing and uncrossing legs or arms, spinning around, etc.

Choose a child within the circle and discuss whether they will be sitting or standing when their turn comes.
Q. How do you know?

Start the pattern again at a different point in the circle. Introduce the word 'predict' and encourage the children to predict whether the 5th child in the circle will be sitting or standing. What about the 10th?
Q. How did you work it out? What information did you use?

Move the group to a larger space where there is freedom to move.
Q. Can you make a shape with your body?

Encourage children to use mathematical vocabulary about shape and position.

Ask children to describe somebody else's shape using our mathematical words. Next ask the children each to make a different shape, remembering to use all the space around them.
Q. How is your shape different from or the same as others in the room? Can you explain why?

  Ref: DfES 1387-2005 G

Encourage the children to all stand very tall, up on tiptoe, arms, hands and fingers reaching to the sky. Explain that this is the first shape in a pattern. Now ask the children to stretch their arms out and their fingers wide and to crouch down low. Explain that this is the next shape in the pattern.

Ask them to make the first shape, then the second, and so on.
Use a drum or a tambourine to develop listening skills and provide a sense of order to the pattern.

Q. Can anyone think of another shape to add to our pattern?
Encourage one child to demonstrate a shape and another to describe the shape they have made.

Now ask all children to form the first shape on the first bang of the drum and hold it for a few seconds. On the second bang children move into position for the next shape, and then finally on the third bang children move into the last shape.

Q. How many different shapes have we created?
Q. Is this a repeated pattern?
Q. How could the pattern become a repeated pattern?

Take responses from the children.

Q. Can anyone describe to us how our three movements could become a repeated pattern?
Ask pairs to discuss how they could change the three movements into a repeated pattern of movements. Choose one pair to demonstrate.

Opportunities for explanation and reasoning here will assess children's understanding of 'repeated patterns'. Allow children time to reflect before they respond.

Q. How many movements did the pair make in total? What was the first shape they made? The second? The third? Then what happened?
Q. How many times did they repeat the first sequence of movements?

Ask children to make up their own sequence of two or three movements and to repeat it, forming a pattern. Ask several pairs to perform their patterns.
Q. How many different shapes were in their pattern? Which was first? Second? Third? And then what did they do?

Activity 3

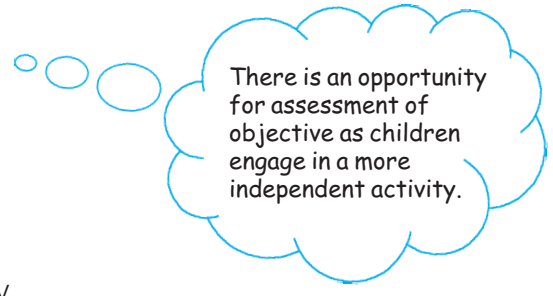
Encourage more-able children to use size as well as colour or to have three rather than two different bricks in their patterns.

If the children find this too easy, ask one child to remove two blocks from different places in the pattern.

Give each pair or group of children construction equipment or interlocking cubes/bricks of different colours and sizes. Ask the children to look at the individual cubes/bricks and describe them to each other.

Q. Can you create a line of blocks which shows a repeating pattern?
Q. Can you describe your pattern to somebody else at your table?

Ask one child to close their eyes while their partner removes a block. The other child should work out which block has been moved and explain why. Ask children to take turns.

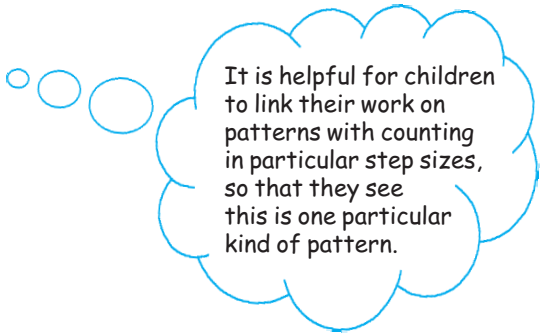


Activity 4

Use an example of a number sequence that the children have been practising, e.g. counting in 2s from 0 or 1, counting in 5s or 10s. Using flashcards on a washing line, ask the children to close their eyes and turn over two or more numbers in the pattern. Ask children to record on their whiteboards the missing numbers in order.

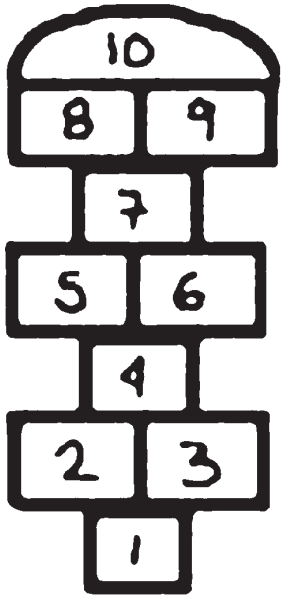
Q. Does this pattern continue forwards and backwards?
Ask the children to close their eyes again. This time include a deliberate mistake in the sequence, e.g. 10, 20, 30, 50, 60, 70.
Q. What is wrong with this pattern of numbers?

Ask the children to work in pairs to write their own sequences of numbers with one deliberate mistake. They should ask another pair to spot the mistake.



Finding rules and describing patterns — Year 2/Year 3

Hopscotch Grid



Resources

- Hopscotch grid
- Digit cards

Objectives

- Solve mathematical problems or puzzles, recognise and explain patterns and relationships, generalise and predict. Suggest extensions by asking 'What if…?' or 'What could I try next?'
- Describe and extend simple number sequences
- Investigate a general statement about familiar numbers or shapes by finding examples that satisfy it

By the end of the lesson, children will be able to:

- describe the rule of a pattern or relationship in own words or pictures;
- test predicted terms to see if a possible rule works;
- use pattern to work out the 10th term in a sequence.

Vocabulary

pattern number sequence increase first, second, tenth, etc.

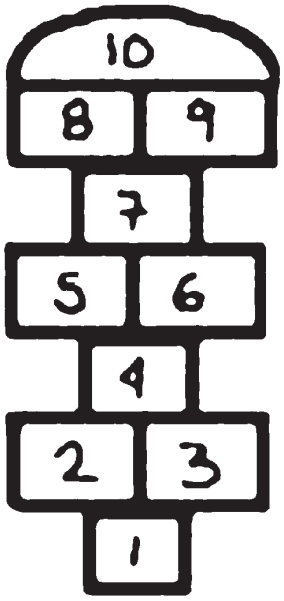
Necessary prior knowledge

Counting in 3s and 5s
Odd and even numbers

  Ref: DfES 1387-2005 G

Main teaching activity

Take your class outside to a hopscotch grid, or draw a hopscotch grid on the board.



It is helpful if children have jotters or rough books available.

This paired discussion will involve more children and allow them time to practise explaining their ideas before contributing to the whole-class discussion.

Q. What number patterns can you see?
Allow children time to discuss in pairs before taking feedback. Jot down all the number patterns they have spotted. Be prepared for them seeing patterns that you have not spotted yourself!

  Ref: DfES 1387-2005 G

Some children may find it helpful to list the single square numbers in order to help them see the pattern of odd and even numbers.

There is enough material in a grid like this to span more than one lesson.

Encourage children to describe the patterns and the rules that they noticed. The sorts of questions you could ask could include:

Q. What is happening to the numbers in the single squares/left-hand double squares/right-hand double squares?

Q. Can you say what the rule is?

Q. Can you continue the sequence in the single squares? In the left-hand double square? In the right-hand double square?

Q. Will the 10th number in the single-square sequence be odd or even?

Q. Can you predict what the 10th number will be in the right-hand double-square sequence?

Q. Why is it easier to predict this 10th number than the 10th number in the other two sequences?

The suggested questions are not a definitive list. You will need to pursue questions that build on the patterns and relationships that your children spot.

Encourage children to sketch the grid to test their hypotheses.

Ask children to continue working in pairs to discuss, explain and record the patterns they find.

Drawing together

Encourage the children to discuss the strategies that they have used to predict numbers, e.g. 'I knew the sequence was the 3 × table, so I knew the 10th number would be 10 × 3', 'I noticed that the numbers in the 2nd and 4th single squares were even numbers so I knew that the number in the 10th square would be an even number'.

Assess children's understanding by asking questions such as 'How do you know?' and 'Why do you think that?' Give them time to explain a range of patterns.

Q. Would number 14 be in a single square? Would 17 be in a double square? Explain to your partner how you know.

It is important that children have opportunities to rehearse explaining their reasoning orally before they are expected to record their reasoning in writing.

Some children will be able to do this by working with the odds and evens and number patterns; others will need to sketch the extension to the grid.

  Ref: DfES 1387-2005 G

Q. What if the grid started at 3?

Explore this idea of 'What if…?' statements with the children and give them time to create some of their own. They can explore grids with different starting numbers.

Here you are giving support to children so that they can ask their own 'What if…?' questions.

Plenary

If children have not spotted that these numbers increase by 5 each time, after some discussion, you could ask them to describe the pattern of numbers, and ask what numbers will need to go in between.

Q. Can you design a hopscotch grid made up of single and double squares so that 6, 11 and 16 are in single squares? Convince your partner.

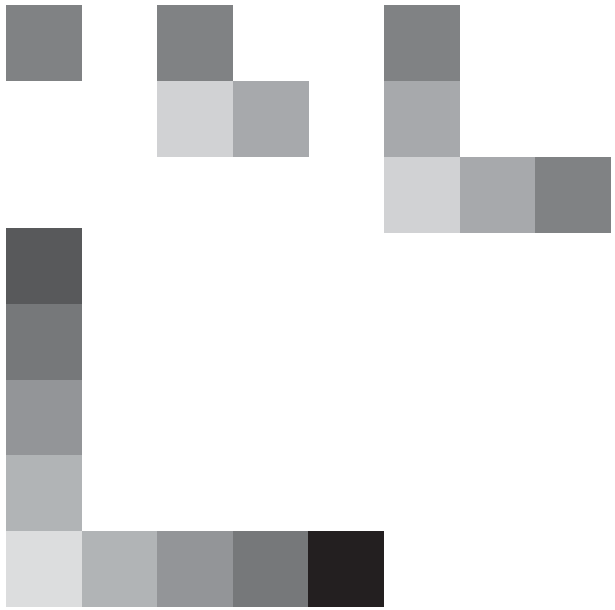
Using questions like these creates good opportunities for you to assess children's understanding of the number patterns they have been working with.

Give children time to rehearse explaining their thoughts to their mathematics partners and then choose some to explain their thoughts to the class.

This rehearsal should help children to clarify thoughts and so improve the quality of their explanations to the class.

Finding rules and describing patterns — Year 3/Year 4

L-shaped models



Objectives

- Solve mathematical problems or puzzles, recognise and explain patterns and relationships, generalise and predict. Suggest extensions by asking 'What if . . . ?'
- Recognise, describe and extend a pattern

By the end of the lesson, children will be able to:

- describe the rule of a pattern;
- predict the 10th term in a pattern.

Vocabulary

pattern repeating pattern same different
increase bigger first, second, etc.

Resources

- Interlocking plastic cubes
- Squared paper

Necessary prior knowledge

Some experience of recognising and describing patterns

  Ref: DfES 1387-2005 G

Main teaching activity

This activity could form an oral or mental starter.

Begin the lesson by clapping a simple rhythm, e.g. 2 quick, 1 slow. Ask the children to join in. Discuss with the children the pattern of the claps and suggest ways of recording it:
e.g. -- _ -- _ -- _ or 22, 1, 22, 1, 22, 1.

Arrange the children so they can work in pairs, with individual whiteboards or jotters.

Clap another simple pattern and ask the children to listen very carefully. Ask children to work in pairs to copy the pattern, and then to record it.

Choose some examples of the recordings to share with the whole class; look for examples that show a variety of pictorial representations.

Ask children to work in pairs to make up a pattern and then record it. Allow the children time to explore, share and agree their ideas.

How is paired work supporting the children?

Circulate around the class and observe how the children approach the task.

Drawing together

Most children will be familiar with graphical scores from their music lessons, so it will be very useful to make links here.

Share some of the children's recordings and ask everyone to clap the patterns.

Now focus on visual patterns, using repeated sequences of shapes, such as

Describe the pattern using vocabulary such as first, second, repeat, etc. Discuss repeating patterns and how they can be extended.

  Ref: DfES 1387-2005 G

Ask children to work in pairs, to create repeating patterns of shapes on their whiteboards. Allow children time to share and develop their ideas.

Q. How can you and your partner describe your pattern?

Ask children to practise describing their patterns to their partners before describing them to the class.

Rehearsing descriptions/ explanations in pairs involves more children and gives them the opportunity to improve their vocabulary. It also can help them feel more confident in front of the class.

Drawing together

Discuss with the class the variety of patterns and the different ways of representing them, drawing on the examples from the lesson and from their previous experiences.

Pursue the conversation to assess the children's understanding using questions such as 'Can you describe…?' 'Why do you think that…?'

Now display some L shapes made from interlocking plastic cubes:

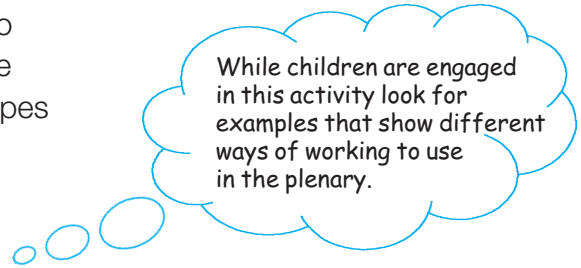
This activity links to the *Speaking, listening, learning* guidance: Speaking Y3 T3 – 'to sustain conversation, explaining and giving reasons for their views and choices'.

Q. What might the next L shape in the sequence look like?
Ask children to discuss this in pairs, come to an agreement and make the next L shape.
Q. How many cubes might be in the next L shape?
Q. How are the L shapes growing each time?

  Ref: DfES 1387-2005 G

Make sure that there are enough cubes available for the children and some squared paper for recording.

Ask the children to work with their partner and use the squared paper to record what is happening to the L shape each time. Remind them of the different ways they recorded the clapping patterns, using symbols, shapes and numbers.



Q. How can you record the way the L shape grows, using numbers?
Q. Would it be useful to create a table?
Q. Would it be helpful to draw the L shape each time?

Ask children to continue finding the number of cubes in the sequence of L shapes. Encourage them to predict the number before making each shape.
Q. Can you predict the number of cubes in the 10th L shape?

Plenary

Use children's work to look at the number pattern that has been created.
Q. What patterns can you see in the numbers you have written down?
Q. Can you explain to your partner what the rule is?

The children should have spotted that the pattern is the sequence of odd numbers.
Look at the sequence of odd numbers on a number line; ensure that children can articulate that the sequence starts at 1 and increases by two each time.

Q. Do you need to make or draw the L shape to predict the next number in the sequence?

  Ref: DfES 1387-2005 G

Q. Can you and your partner make a shape that would grow in the sequence of even numbers?

You could challenge the children to investigate other letter shapes such as T and E in this lesson or in subsequent lessons. Children could use cubes or squared paper and record their results in tables.

Allow children time to discuss and investigate.

Q. Can you and your partner explain to each other why you must start with an even number of cubes?

Q. Can you use what you have learned today to predict or work out how many cubes there would be in the 10th shape?

Agree that if the pattern is the sequence of odd numbers, then the 10th shape will have the 10th odd number of cubes.

Draw the following table on the board:

Shape	1	2	3	4	5	6	7	8	9	10
Number of cubes	2	4	6	8	10	12				?

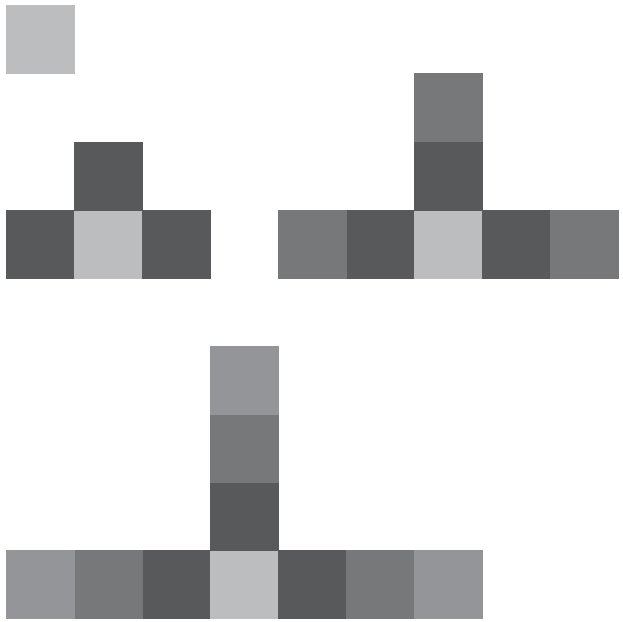
Q. This pattern is the sequence of even numbers. Is there a quicker way of finding the number of cubes in the 10th shape than counting in 2s?

Encourage children to see the relationship between the number of the shape and the number of cubes in it.

  Ref: DfES 1387-2005 G

Finding rules and describing patterns — Year 5/Year 6

Sequence of models



Objective

- Solve mathematical problems or puzzles, recognise and explain patterns and relationships, generalise and predict. Suggest extensions by asking 'What if . . . ?'

By the end of the lesson, children will be able to:

- accurately predict a later term in a pattern or sequence;
- express the general term in words and begin to do this algebraically.

Vocabulary

pattern sequence term difference general term predict

Necessary prior knowledge

Experience of describing and extending patterns
Some experience of recognising the relationship between a term and its position in a sequence

Resources

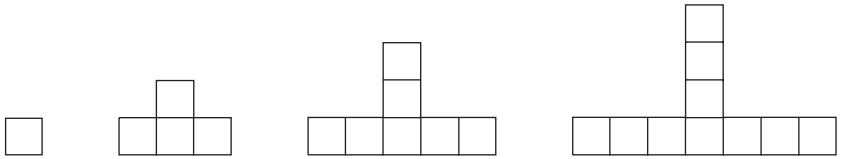
- Individual whiteboards
- Interlocking plastic cubes
- Squared paper

  Ref: DfES 1387-2005 G

Children who are working significantly behind their year group may benefit from the Year 3/Year 4 lesson, which uses a different pattern of interlocking plastic cubes.

Main teaching activity

Introduce the purpose of the lesson: to describe in words and symbols the general term in a shape pattern.
Show the children these four models made of interlocking plastic cubes in a pattern, ready made.



It is important that children do not see a 2-D representation until later in the lesson.

Ask children to build their own four models.

Making the models helps children to see how each is different.

Q. How is the pattern developing?
Ask children to explain to their partners how they see it developing.
Take class feedback.
Q. So how exactly do you get from one to the next?
Ask children to discuss this in pairs. Encourage children to use precise mathematical language.
Q. Do you both see it developing in the same way? Can there be more than one way of seeing it?
Q. So can you now make the 5th in the sequence, the 5th term? Show what you have made to the person next to you. Have you made the same one? Did you do it in the same way? How did you know what to do?
Q. Could you make the 7th term? Compare yours with the person next to you. How did you do it? Was it harder than making the 5th term?
Highlight how the 7th term is harder because it is not the next term.
Q. Would you have an idea now how to make the 12th term?
Ask children to discuss this in pairs.

This paired discussion helps children understand the problem and see it in different ways.

  Ref: DfES 1387-2005 G

Q. How could we record our pattern on squared paper so that we show the way that the pattern is developing?

Draw out, using colour or shading. This is one possible way of doing it.

It is important here to help children make the transition from the concrete to the visual recording on paper.

Discuss strategies children are using to show how they see the pattern developing.

Q. Let's discuss the number of cubes in each part of the pattern. This will help us with devising a general term. What do we mean by a general term?

Encourage children to come up with a definition in pairs. Let one in each pair record on whiteboard.

Agree on a class definition.

Observe how the paired work supports the children's development of the concept.

Suggest that a table will help us organise our thoughts.

Term	1st	2nd	3rd	4th	5th	
Number of cubes	1	4	7	10	13	
How pattern develops						

Invite children to show you on whiteboards how they see the pattern developing with the numbers.

Here are some suggestions of the ways that children might see the pattern developing.

Term	1st	2nd	3rd	4th	5th
Number of cubes	1	4	7	10	13
Explanation A	1	1 (previous number) + 3 (new ones)	4 + 3	7 + 3	10 + 3
Explanation B	1	3 (horizontal) + 1 (on top)	5 + 2	7 + 3	9 + 4
Explanation C	1	1 + 3 (round the sides)	1 + 6	1 + 9	1 + 12
Explanation D	1	1 + 3	1 + 3 + 3 (three new ones)	1 + 3 + 3 + 3	1 + 3 + 3 + 3 + 3

This may need more work if children have not met the use of *n* before.

Discuss with children that Explanation A, where the next term is connected to the one before, doesn't help us to find a general term. It only helps us to find the next one if we know the previous one.

Stress that we want to find a way of finding any term if all we know is its position in the pattern.

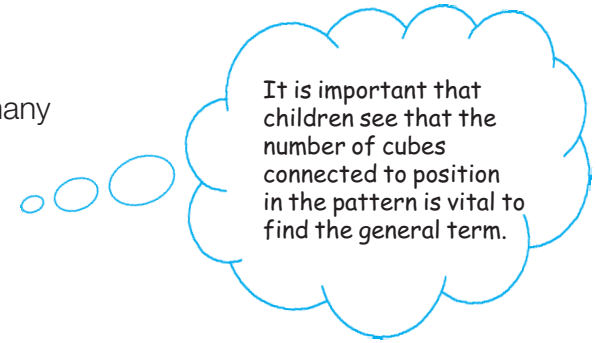
  Ref: DfES 1387-2005 G

This assumes that children are familiar with multiplying out brackets.

Explanation B, which looks at the horizontal and vertical cubes, again doesn't help connect to the position of the model in the pattern.

Q. How could Explanation C be developed? How do we know how many 3s to add?

Agree that this is the term number less 1, i.e. $n - 1$.



Discuss Explanation D.

Q. Can you see how the number of cubes in the model connects to the term number?

Always start with one and add 'one less lots of three' than the term number.

If we call the term number n then this could be written as $1 + 3 \times (n - 1)$ or $1 + 3n - 3 = 3n - 2$.

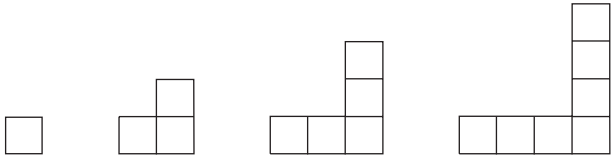
Q. Does this work? What happens if the term number is 3? 4? 5? Do we get the number of cubes that we would expect?

Children who use more of the numbers may describe the general term as $3n - 2$. Draw out that this is the same and we can still use the diagrams to explain it, because there are n groups of cubes, but one group only has one cube, not three, so we subtract two.

Plenary

This pattern is used in the Year 3/Year 4 lesson.

Try a different pattern and ask children to work out what the general term is.
Use the L shape. Draw the pattern on the board.
Q. How do we see the pattern developing?
Ask children to discuss this in pairs.



Take feedback.
Q. If we want to find a general term what do we need to do next?
Put the term numbers on the board.

This plenary provides an opportunity to see how much the children have grasped.

Q. What does the grid look like?
Ask children to discuss, then draw ideas on the board, as a class.

Shape	1st	2nd	3rd	4th
Number of cubes	1	3	5	7
Pattern				

Q. Can we see how the pattern develops connected to the term number?
Ask children to show their answers on their whiteboards.

  Ref: DfES 1387-2005 G

Invite children with different suggestions on their whiteboards to come to the front and hold them up for the class to discuss.

Q. Which do we think is most helpful to find the general term?

Q. Why do we think this is the most helpful? Convince your partner.

Term	1st	2nd	3rd	4th
Number of cubes	1	3	5	7
Pattern	1	1 + 2	1 + 2 + 2	1 + 2 + 2 + 2

Discuss ways in which children convinced their partners.

Q. Can we write the general term using the letter *n* for the position in the pattern? Ask children to show their answers on their whiteboards.
$1 + 2(n - 1)$ or $2n - 1$

Q. Does your answer work if *n* is 4, i.e. looking at the 4th term?

Ask children to tell the person next to them what they have learned today. Draw out that to describe a general term, we need to relate it to its position in the sequence, not just to the previous term.
Although we may see the sequence differently we will end up using the same general term using mathematical notation.

  Ref: DfES 1387-2005 G