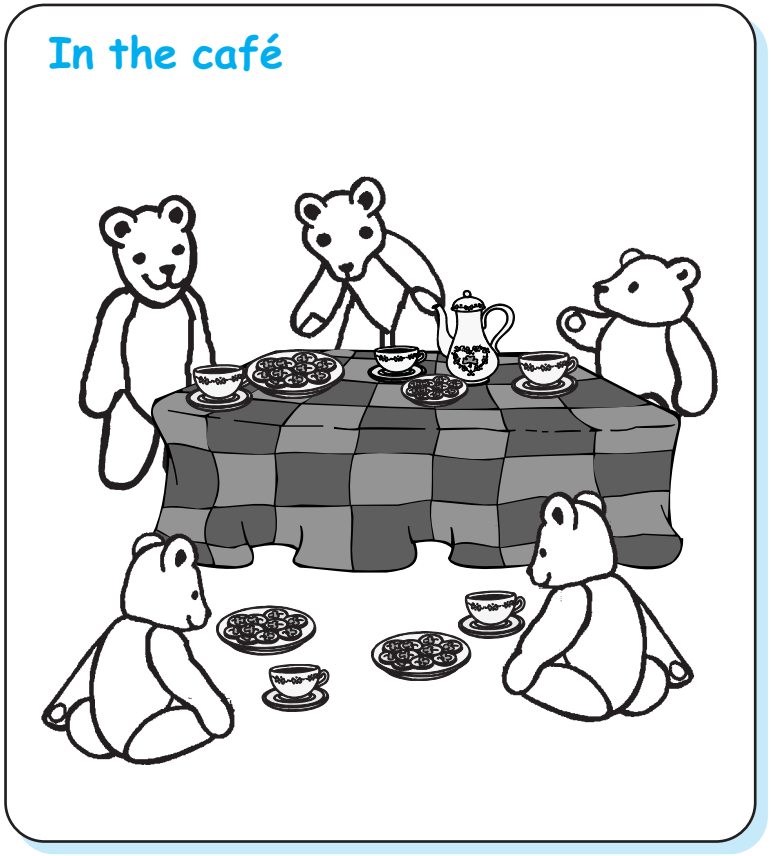
Finding all possibilities — Foundation Stage — Activity 1



Resources

- Teddies or other soft toys
- Coloured crockery and cutlery

Early Learning Goal from *Curriculum guidance for the Foundation Stage:*

- Use developing ideas and methods to solve practical problems

Objectives from *NNS Framework for teaching mathematics from Reception to Year 6*

- Solve simple problems or puzzles in a practical context, and respond to 'What could we try next?'
- Sort objects justifying the decisions made
- Use developing mathematical ideas and methods to solve practical problems involving counting and comparing in a real or role-play context

By the end of the lesson, children will be able to:

- recognise similarities and differences;
- sort objects, using a number of criteria;
- explain what they are thinking and doing.

Vocabulary

same sort different set

Necessary prior knowledge

Colour recognition
Language of similarity and difference
Experience of playing in a 'house' situation

  Ref: DfES 0545-2004 G

Role-play activity

Suggestion for a supportive activity:

Put a selection of the crockery and cutlery into a feely bag or box. Get a child to take out an item and describe it, e.g. 'It's a saucer that's red and round.' Repeat. Extend to two items and discuss the similarities and differences, e.g. 'They're both cups, but that one is red and the other is blue.'

Establish that the children understand the concept of setting/laying the table.

You may want to say that they are going out for a meal in a pub, café or restaurant.

It may be helpful to add to the real-life context by reading stories that model the problem, e.g. 'The Teddy Bears' picnic.'

You may notice that some children match the colours systematically. 'I've given that teddy all the green ones because he's got a green tie on' or 'He likes red things best.'

Within role-play situations there are many opportunities for us to encourage the children to develop their problem-solving skills. This activity is in the context of role-play in the home or café, and involves the children in preparing the table for the teddies to have a meal. There will need to be a collection of soft toys and coloured crockery and cutlery from which they can choose. The adult's role may be as an observer of the children's play or 'in role' with the children.

Children need to understand that problem solving involves choices so they have the opportunity to make decisions and justify them.

You may need to ask some of the children questions to get them started.

Q. Which teddies are having tea today? How many are there?
Q. Do you think there are enough plates for all the teddies to have tea?
Q. How can you be sure each teddy has a plate?
Q. How are you going to start setting the places?

Observe how the children tackle the problem.

Encourage the children to discuss with one another how the settings are the same or different.

Creating a 'have a go' climate encourages positive learning.

Q. What do you notice about the plates you have given the teddies?
e.g. Oh yes, those two teddies have both got a blue plate.
Yes, they are all different.
Q. What about the cups and saucers? Do they match? Are they different?

Create related problems and ask questions using other criteria to extend the children's thinking.

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Q. Can you think of different ways of doing it?
Q. I think this teddy doesn't like red things. What can you do to help him have a setting he likes?
Q. Both these teddies want a yellow knife and fork. Can you sort that out for them?
Q. Can you make completely different settings for two teddies?
Q. These two teddies both want place settings that are only green and red. Can you make them both different?
Q. What would you do if another teddy came to the table? Is there enough room? Have you got enough cups?

In the context of a related story or later discussion with a larger group of children about what they have been doing, it may be appropriate to model what some of the children did and draw out discussion about same/different.

How did the children tackle the problem? Did they understand the problem? Did they use a random approach? Did they use the words 'same/different'? Did any of them give each teddy all the same colour crockery and cutlery? Were they able to talk to others, including you, about what they were doing?

Encourage the children to talk to one another about what they are thinking and doing.

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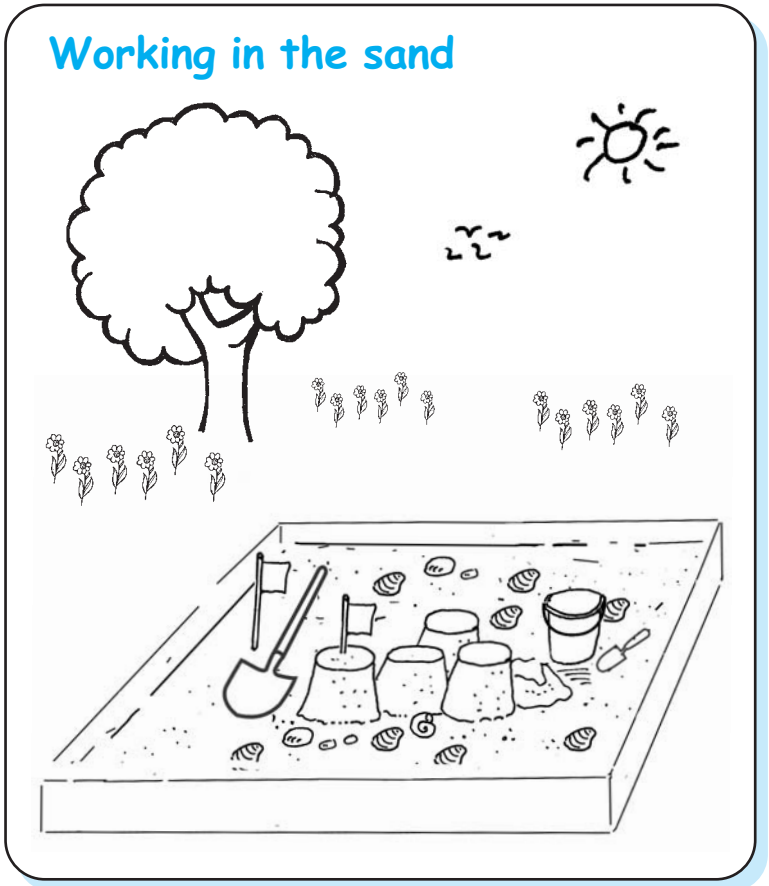
Finding all possibilities — Foundation Stage — Activity 2



Resources

- Sand tray
- A collection of objects to hide in the sand

Early Learning Goal from *Curriculum guidance for the Foundation Stage*:

- Use developing ideas and methods to solve practical problems

Objectives from *NNS Framework for teaching mathematics from Reception to Year 6*:

- Solve simple problems or puzzles in a practical context, and respond to 'What could we try next?'
- Sort objects justifying the decisions made
- Use developing mathematical ideas and methods to solve practical problems involving counting and comparing in a real or role-play context

By the end of the lesson, children will be able to:

- recognise similarities and differences;
- give examples that match a given statement and ones that don't;
- explain what they are thinking and doing.

Vocabulary

same	sort	how many?	order
different	set	size	match

Necessary prior knowledge

Language of similarity and difference

  Ref: DfES 0545-2004 G

Suggestion for a supportive activity

Use a brightly coloured box as a 'Magic Box' and a collection of pairs of objects that have similarities and differences. Initially, take out an object and get the children to describe it. 'Yes, it's a big green car.'

Tell the children that if you put something into the magic box it is different in some way when it comes out again. Get them to describe the object going into the box. 'It's a long red sock.' Take out a short red sock and ask, 'What's changed?…Yes, it's a short sock now.' Put in a cup and take out a mug, or a yellow triangle and take out a yellow square. Repeat with similar objects which appear to change colour or size.

For some children, it may be appropriate to extend the activities by hiding wooden or plastic numerals in the sand, giving children the opportunity to find, sort, match and also to order the found numbers.

Sand tray activity

This activity extends the use of the sand tray and may be done indoors or outside. You will need to decide whether the adult is to be an observer of how the children tackle the problem or an active player and prompter alongside them.

Put a collection of objects in the sand for the children to find and sort. The choice of objects may be linked to other learning in a variety of ways, for example, a current story (The Three Bears – teddy counters), an appropriate theme (The Zoo – animal counters), the role-play area (The Fruit Shop – plastic fruits) or another area of the classroom learning environment (Construction area – bricks, Autumn display – laminated leaves).

Hide the objects for the children to find as they play in the sand.

Q. I've lost some things in the sand tray. Can you find them for me? Some are the same and some are different but they'll all be mixed up. See if you can sort them out.

It may be a good idea to have a collection of empty containers beside the sand tray into which the children can place the found objects as they sort them.

If you are playing with the children, model showing one another what has been found and describing the objects using the vocabulary of same/different. 'Oh look, I've found a yellow banana too. It's the same as the one Jed found. Is it the same as your fruit, Kelly?'

Perseverance and persistence are important problem-solving skills. 'I hid 10 in the sand. How many have you found? How many more do you need to find? Can you find the rest?'

Comparing two objects which have similarities and differences will help children to realise that there is more than one way to sort the collection of objects.

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The children may have difficulty noticing objects which have similarities and differences, and may need your prompts: 'Yes, they are both the same because they're both apples, but what's different about them? Yes, Rashid, yours is a red apple and Lee's apple is green.'

Encourage the children to compare the objects using different criteria such as colour, size or type, noticing similarities and differences.

Q. Lee, is your apple the same as Rashid's?

Q. You two have both found the same kind of creature, they're both snakes, but what's different about them?… Oh, Sally says her snake is longer than Harry's snake. Yes, Harry you're right, you noticed they're different as well because yours is green and Sally's is purple.

Encourage the children to use the empty containers to group the sorted objects and ask them to justify their groupings.

Q. So what's the same about all the things in this bucket?

The need to find all the objects gives the children a reason to count the ones they have found.

Q. How can we find out if we have found them all yet?

Some children may be able to sort the found objects further. They may be able to say, 'All these bricks are blue, but these are all small blue bricks and those are all big ones.'

If you have the use of an interactive whiteboard, you may want to create a screen with a collection of objects and sorting boxes into which the objects can be dragged and dropped.

In a later discussion with a larger group of children about what they have been doing, it may be appropriate to show how the children sorted the objects, drawing out what was the same and what was different.

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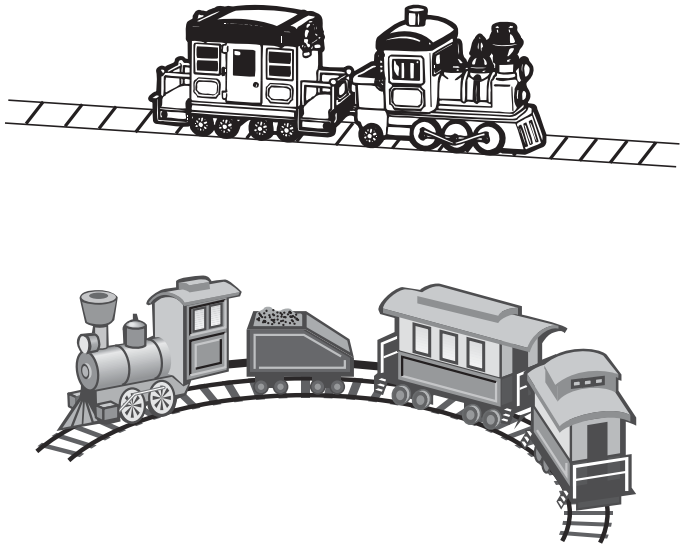
Finding all possibilities — Foundation Stage — Activity 3

Railway track



Resources

- Railway track and other related objects, e.g. trains, station, bridges

Early Learning Goal from *Curriculum guidance for the Foundation Stage*:

- Use developing ideas and methods to solve practical problems

Objectives from *NNS Framework for teaching mathematics from Reception to Year 6*:

- Solve simple problems or puzzles in a practical context, and respond to 'What could we try next?'
- Sort objects justifying the decisions made
- Use developing mathematical ideas and methods to solve practical problems involving counting and comparing in a real or role-play context

By the end of the lesson, children will be able to:

- recognise similarities and differences;
- sort objects, using a number of criteria;
- explain what they are thinking and doing.

Vocabulary

same
different

Necessary prior knowledge

Language of similarity and difference

Mini-world/construction activity

Mini-world and construction provide many opportunities for children to develop their problem-solving skills.

This activity will involve the children making two tracks to get from one point to another. It could be helpful to mark the beginning and end of the track with a station. The children will need a selection of straight and curved track and other related items such as bridges and level crossings. The adult's role may be as an observer of the children's play or as a player with the children. Ask the children to make two different tracks from one station to another.

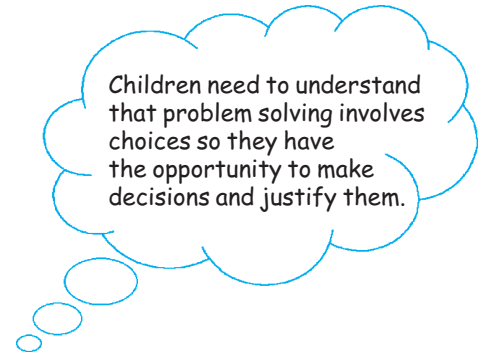
It may be helpful to add to the real-life context by reading a story about railways and trains.

You may need to ask some of the children questions to get them started.

Q. What sorts of pieces of track shall we use?
Q. What shall we put on our tracks?
Q. How can we make them different?

Observe how the children tackle the problem.
Encourage the children to discuss with one another how the tracks are the same or different.

Q. What do you notice about the two tracks you've made?
Yes, that one's got lots of curved pieces and that one's got lots of straight pieces.
Q. Which track is longer?
Q. What else have you put on your tracks?
Q. What's the same about the two tracks?



How did the children tackle the problem? Did they understand the problem? Did they use the words 'same/different'? Were they able to talk to others, including you, about what they were doing?

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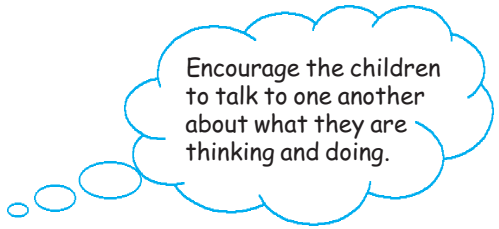
It might be that both are made from wood, or both are made from plastic, or both have a bridge in them, or both have curved and straight tracks. They also start and finish at the same stations.



Create related problems and ask questions with other criteria to extend the children's thinking.

Q. Can you think of how we could make the tracks completely different?

Q. This train doesn't like too many curves. Where could we make some changes?

Q. This train can't go up hills. What might we need to change?

Q. Why might this track need to be longer than the other one? Can we add some extra stations on one track for people who only want to travel short distances? Which would be best? Why?

If you have an interactive whiteboard or projection equipment, you could ask the children to take digital photographs of their track layouts and then choose some to prompt the discussion.

In the context of a related story or later discussion with a larger group of children about what they have been doing, it may be appropriate to sketch the tracks that some of the children made and discuss what is the same and what is different about them.

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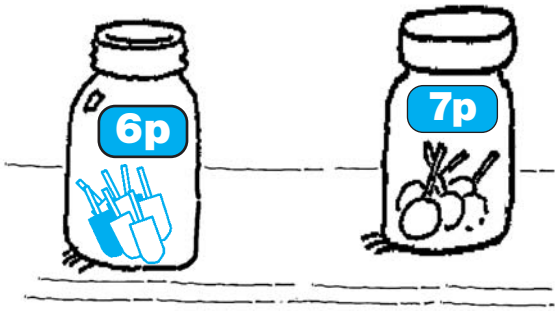
Finding all possibilities — Year 1 — Lesson 1

Lollipops

Jade bought a lollipop. It cost 6p.



She paid for it exactly. Which coins did she use?

There are 5 different ways to do it.
Find as many as you can.

What if the lollipop cost 7p?

Objectives

- Solve mathematical problems or puzzles, recognise simple patterns or relationships, generalise and predict. Suggest extensions by asking 'What if…?' or 'What could I try next?'
- Organise the recordings of possibilities, e.g. in an ordered list

By the end of the lesson, children will be able to:

- recognise that there is more than one possible answer to a problem;
- explain how answers differ and to record different answers in a list.

Vocabulary

pence	buy
total	pay
cost	same
list	different
coin	

Resources

- Resource Sheet
- Jars of lollipops or other items labelled with '4p each', '6p each' and '7p each'
- Puppet
- Mega money (large coins)
- Real coins (1p, 2p and 5p)
- Strips of card with double-sided tape stuck on the back
- Sheet of A2 paper (optional)

Necessary prior knowledge

Recognise coin values and understand the order of the values
Add more than two numbers, 5 or less
Count in ones and twos

Main teaching activity

Another good starting point might be based round the book *The Great Pet Sale* by Mick Inkpen, using animals as the items to be purchased. This could link literacy and mathematics.

The class shop could form a focus for other problem-solving activities.

Set the context for the problem by showing the class several jars of lollipops with '4p each', '6p each' and '7p each' labels or items from the class shops, similarly labelled. Explain that anyone who wants to buy a lollipop from the jar must pay the exact amount for it.

Introduce a puppet named Jade (or any familiar class puppet). Explain to the children that they are going to solve problems today to help Jade to buy some lollipops.

If you have an interactive whiteboard with software which allows large coins to be displayed and moved, this would support the manipulation of the coins and the recording.

Draw the children's attention to the jar labelled '4p each'.

Q. If Jade wants to buy one lollipop out of this jar how much money will she need?

Identify that she will need coins. Agree that she needs 4p.

Q. What will Jade use to pay for the lollipop?

Q. What coins might she use?

As the children respond, put large 1p and 2p coins on the board.

Q. Why wouldn't she use a 5p, 10p, 20p or 50p coin?

Agree that these are more than 4p.

Some children will not be aware that there is more than one answer. It is important for them to understand that the cost is the same, but that you are using different coins to pay and there are different ways to do this.

If the children only suggest 1p coins, hold up a 2p coin and ask how much more money is needed to make 4p.

Invite a child to come and give you 4p for a lollipop. Show the children how to record the solution, for example by drawing four 1p coins (or displaying mega money coins), and recording:

$1p + 1p + 1p + 1p = 4p$

If children suggest examples where giving change is involved, remind them that in this situation they have been asked to find the exact amount.

Stress the importance of checking the total, for example by keeping a tally on your fingers.

Q. Is there another way to pay for a lollipop?

Ask the children to work in pairs to find different ways to make 4 pence.

Drawing together

Draw the class together to share solutions.
With the children check that no answer has been repeated.
Q. Why are these answers different? Why are these answers the same?

To help the children 'see' the answer, attach the coins to the board, then record the number statement alongside.

If children present an answer that involves the same coins in a different order, rearrange the coins using mega money for example, to show the equivalent solutions.

Draw out that 1p + 1p + 2p uses the same coins as 1p + 2p + 1p and 2p + 1p + 1p.

Make a list of the solutions:
1p + 1p + 1p + 1p = 4p
1p + 1p + 2p = 4p
2p + 2p = 4p

Helping children to recognise the same and different answers helps them to be more systematic. Making a list is one way that children can be helped to see whether all possibilities have been found.

Tell the children that you have recorded the different answers in a list.
Q. How many different answers were there to this problem? Are there any more answers?
Agree that there are three answers.

Draw the class's attention to the jar labelled '6p each'.
Q. If Jade wants to buy a lollipop out of this jar, how can she pay?
Q. How many different ways do you think Jade can pay? Why?
Explain that Jade wants to know all of the different ways she would be able to pay for the lollipop.
Ask everyone in the class to find a way to make a total of 6 pence.
Explain that the class will regroup to make a list of the different answers they have found.

Pursue the conversation to assess the children's understanding using questions such as:

'How do you know that?', 'Why do you think that?'

On an A2 sheet of paper write the title 'Ways to make 6p'. Explain that you will record the list of all ways to make 6p.

Ask the children to each find one way to make 6 pence using the real coins. Encourage them to find a way that is different to their neighbours'. When they have laid out their 6 pence amounts they should stick them onto a strip of card with double-sided sticky tape as a record.

Plenary

Ask the children to bring their strips of card and coins to the plenary. Remind the children that the problem was to find all of the ways in which Jade could pay 6p for her lollipop. Say that they now need to make a list of the different ways and check whether the answers are the same or different.

Invite a child to put their solution onto the paper.

Q. Does this equal 6p? How can we check?

Invite other children to add their solutions to the list. Each time ask children to check that the amount equals 6 pence and that it is different to the other solutions.

Q. Does this solution equal 6 pence?

Q. How could we change it so that it does equal 6 pence?

Q. Have we had this solution already? How do you know?

Q. How is this solution different to this one?

It is not necessary for the children to draw the coins or write the amounts. Sticking the coins onto strips is an appropriate record. You will model the written recording at the end of the lesson.

Observe how children approach the task and the most common difficulties, and identify children who quickly found a solution.

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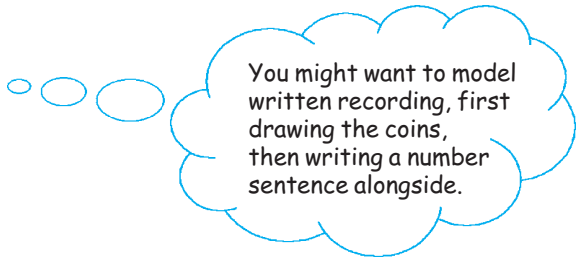
Show how to make a written record of the five solutions:

1p + 1p + 1p + 1p + 1p + 1p = 6p
1p + 1p + 1p + 1p + 2p = 6p
1p + 1p + 2p + 2p = 6p
2p + 2p + 2p = 6p
1p + 5p = 6p



Conclude the lesson by referring back to the original problem.

Q. How many different ways of paying 6p for the lollipop did we find?
Q. Which way uses the most coins?
Agree that it is when we pay with 1p coins only.
Q. How many 1p coins did we use?
Agree that it was six.
Q. Which way uses the fewest coins?
Agree that it is when we use a 5p coin and a 1p coin.
Q. If the lollipop cost 8p, how many 1p coins would we use?
Q. Could we use 2p, 5p or 10p coins?
Agree that 2p and 5p coins could be used but 10p is too much.
Q. If we used a 5p coin, how much more money is needed? What coins could we use to make that amount?

'What if…?' questions provide useful extensions to problems.

'If Jade wanted to buy a lollipop out of the 7p jar, would there be more answers?' extends children's thinking at the end of this problem.

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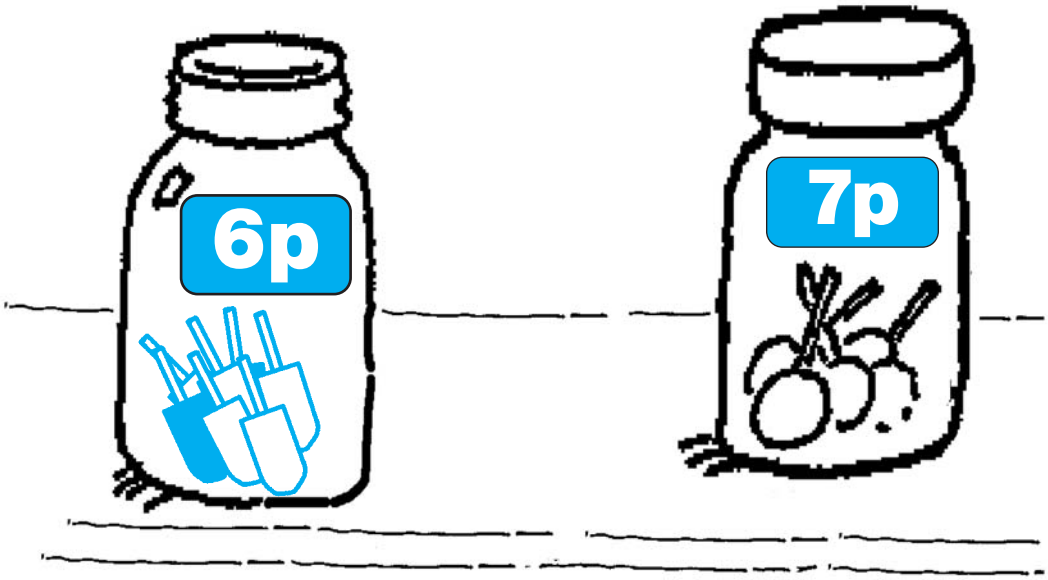
Resource Sheet

Lollipops

Jade bought a lollipop. It cost 6p.



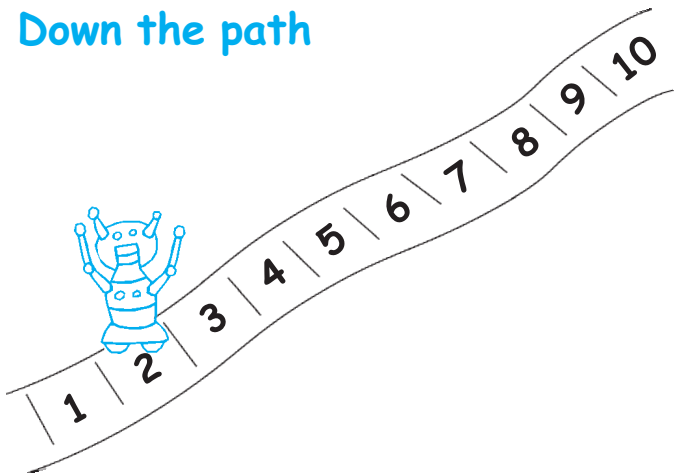
She paid for it exactly. Which coins did she use?

There are 5 different ways to do it.
Find as many as you can.

What if the lollipop cost 7p?

Finding all possibilities Year 1 Lesson 2

Down the path



The robot is on 2.
You roll a 1 to 6 dice.
After 2 moves it lands on 8.

Find all the different ways the robot can do this.

Now think of other questions you could ask.

Objectives

- Solve mathematical problems or puzzles, recognise simple patterns or relationships, generalise and predict. Suggest extensions by asking 'What if…?' or 'What could I try next?'
- Organise the recordings of possibilities, e.g. in an ordered list
- Begin to have a system for finding the possibilities, e.g. start with the smallest number

By the end of the lesson, children will be able to:

- explain how answers are different and to record different answers in a list;
- begin to have a system for ordering a list of possibilities.

Vocabulary

list
count on

Resources

- Resource Sheets 1 and 2
- Activity Sheet
- 1–10 floor number track
- Moveable robot or object
- Objects for moving along individual tracks on Resource Sheet 1
- Two large demonstration dice
- Individual dice

Necessary prior knowledge

Adding two numbers
Counting on
Pairs of numbers that make 6
Playing a track type game

Using a programmable robot as the context for the lesson could be effective. The steps on the number track would need to be the same size as the steps the robot makes.

Alternatively use a character from the small world or a soft toy and move it by hand. The context for the track might fit in with a topic being covered.

Main teaching activity

Sit the class around a large-scale number track (the path) numbered 1 to 10 and a robot/object to move along it. Give each pair of children a copy of Resource Sheet 1, two dice and an object to move along one of the tracks in front of them. Ensure that the children understand the idea of the game – to roll the dice and to move the object along the track the number of steps rolled.

This is a good opportunity to model recording as a number sentence: 5 + 2 = 7

Q. If the robot is on 5 and I want the robot to move onto 7 next, what number would I now be hoping to roll on the dice?

As they work, observe how well children count on. Intervene to model the language of adding the two scores together to reach a position.

'You rolled a 4 and then you rolled a 3, so 4 steps and then 3 steps meant you arrived at 7.' Questions such as 'What if you wanted to get to 9?' promote imagery.

You may need to repeat this process several times to ensure that the children are used to reaching a target number, rather than counting on the number stated. Some may want to count on another 7 rather than visualise what is needed to get to 7.

Take responses and try them out. Agree that two more steps would take the robot to 7.

Ask the children to work in pairs to find other pairs of numbers that would enable the robot to move from the start to 7 in two moves. Ask them to turn their dice to show the two numbers that might be rolled.

Drawing together

Q. What two numbers did you turn your dice to?

Model the recording, e.g. 3 + 4 = 7

Q. What is the biggest number you could use? What would you need to get next?

Record 6 + 1 = 7

Q. What's the second biggest number you could use? What would you need to use next?

It is important to make connections to previous lessons. You may want to show the list from yesterday here.

This is a good opportunity to model systematic working.

Record 5 + 2 = 7
Continue until all the solutions are recorded.
Discuss that 2 + 5 = 7 and 5 + 2 = 7 are similar but the order that the numbers come up on the dice is different.

Some children might be able to record their two moves as a number sentence.

Give out Resource Sheet 2 and explain that the robot is now starting on the number 2 and you want it to get to the number 8, but it must be done with two rolls of the dice. Explain that there is more than one way to do this and we need to find all of the different ways. Remind them of the list that you have just made and say that this is a useful way to record the different ways. Show a large version of the Activity Sheet to show how the different possible answers are going to be listed.

You might need to show this visually, e.g. 'So it might move here in the first move, then move to 8 on the second move, or it might move here in the first move and then it has this far to go in the second move.'

Demonstrate by using two large dice, one for the first throw and one for the second throw. Tell the class that the first throw might be a 4 (showing a 4 on the large dice). Set the robot to move forward 4 from 2 to land on 6.

Q. So what would my second throw have to be?

Establish that it would be 2. Show the 2 on the second dice. Test by getting the robot to move forward 2.

Q. How many moves did it take to get to 8?

Establish that 6 moves are needed, and that is what the problem is asking. In other words 'How can we make 6 with two dice?' Say that we could also record this as 4 + 2 = 6.

'What if…?' questions provide useful extensions to problems. An appropriate one here is: 'What if I put a "Go back to the start" label on one of the numbers of the track, which solution wouldn't work now?' 'What number could I put the label on so that this solution wouldn't work' leading into checking the list of possibilities against a criterion.

'Snakes and Ladders' from *Mathematical challenges for able pupils in Key Stages 1 and 2* develops this idea well.

Ask the children to work in pairs to place their object on number 2, and then turn (not roll) their two dice as a pair to agree what their two dice rolls would need to be to get to 8. They should record this solution on the Activity Sheet. They may wish to circle the numbers they landed on, on Resource Sheet 1. You may need to remind them that a total of 6 is needed.

Pursue the conversation to assess the children's understanding such as 'How do you know?…' and 'Why do you think that?'

While children are engaged in this activity, look for examples that show systematic working.

Observe how well children engage with the task in hand. Do they focus on finding a pair of dice scores to total 6? Is paired work successful?

How do they approach the task?

Drawing together

Take children's responses on any pairs they have found. Invite two children out to the front to show two dice scores with the large dice.

Q. If we want to make a list of the different solutions, what would be a good one to start with?

Agree that starting with the smallest or largest number is helpful.

Q. How is this answer different to this one?

Model how to record in a systematic way on the large version of Activity Sheet 1.

Ask the children to work in pairs to find other pairs of dice scores that make 6 and record them on the Activity Sheet.

Plenary

Some children will need to test their solutions on their tracks.

Bring the children together, along with their lists, to see whether different ways to solve the problem have been found.

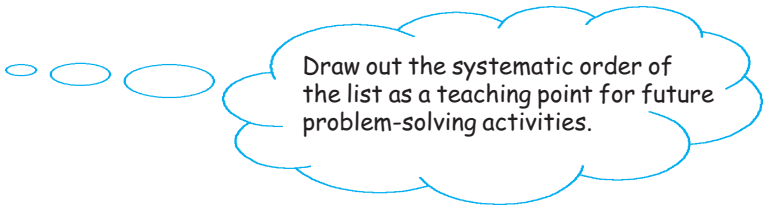
Test answers by moving the robot along the track using the children's answers.

Identify different answers and compile a class list in a systematic order:

1 and 5
2 and 4
3 and 3
4 and 2
5 and 1.

Point out the pairs of answers:

1 and 5 2 and 4
5 and 1 4 and 2



Discuss how the same pairs of numbers were rolled but in a different order.

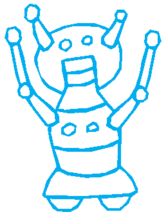
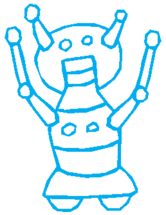
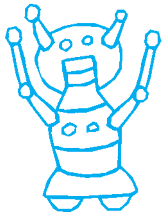
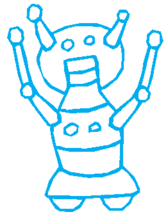
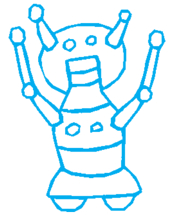
Conclude the lesson by referring back to the problem and agreeing that there are five different ways to solve the problem.

  Ref: DfES 0545-2004 G

Finding all possibilities

Year 1 Lesson 2

Resource Sheet 1

1	2	3	4	5	6	7	8	9	10

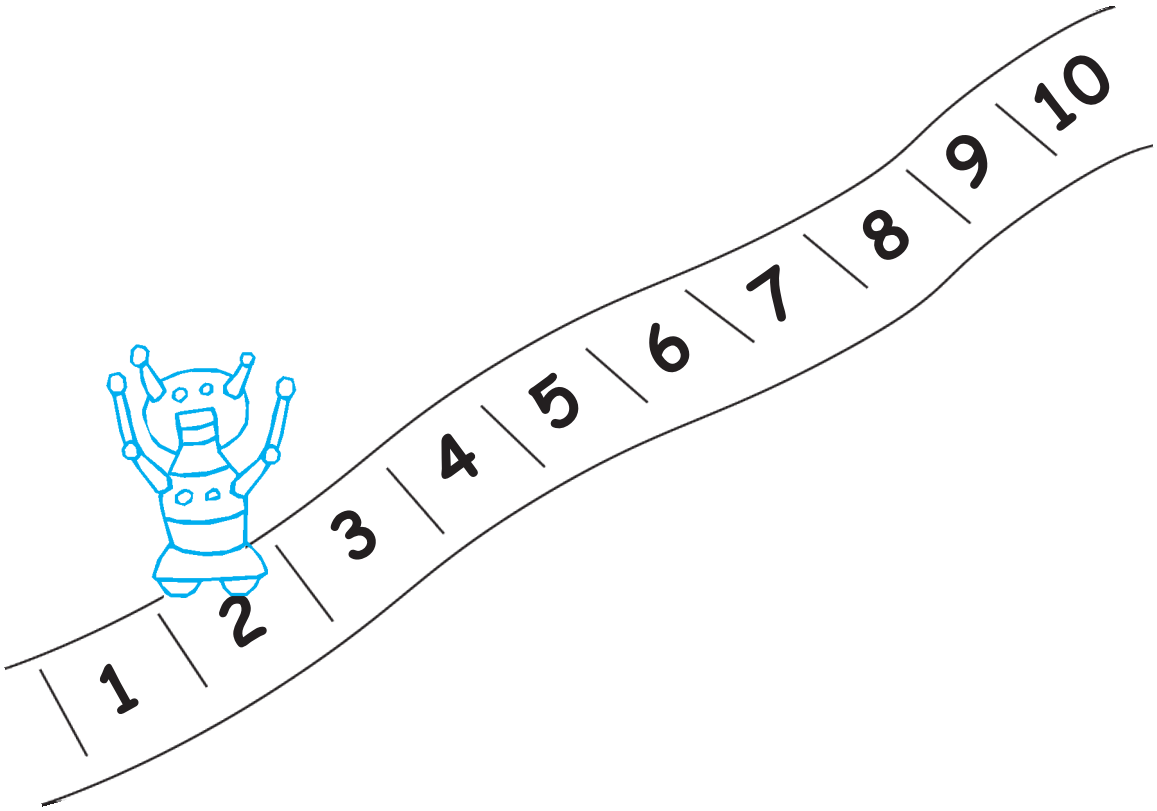
1	2	3	4	5	6	7	8	9	10

1	2	3	4	5	6	7	8	9	10

1	2	3	4	5	6	7	8	9	10

Resource Sheet 2

Down the path



The robot is on 2.
You roll a 1 to 6 dice.
After 2 moves it lands on 8.

Find all the different ways the robot can do this.

Now think of other questions you could ask.

Finding all possibilities Year 1 Lesson 2

Activity Sheet

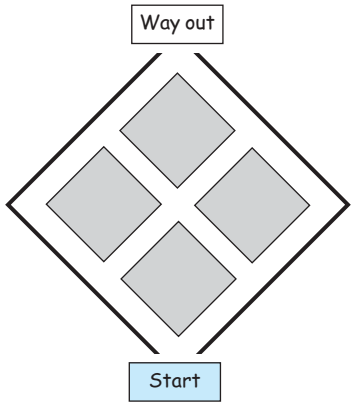
Ways to land on 8, in 2 moves

move 1	move 2

Finding all possibilities Year 2 Lesson 1

Maisie and the maze

Maisie explored the maze.
She always went forward.



How many different ways are there for Maisie to go from the start to the way out?

Resources

- OHT of Maisie and the maze
- Activity Sheet of copies of the maze
- Maze drawn five or six times on the playground with chalk

Objectives

- Solve mathematical problems or puzzles, recognise simple patterns and relationships, generalise and predict. Suggest extensions by asking 'What if…?' or 'What could I try next?'
- Have a system for finding the possibilities, e.g. start with the smallest number
- Organise the recording of possibilities, e.g. in an ordered list or table

By the end of the lesson, children will be able to:

- begin to use a systematic way to solve a problem involving finding all possibilities;
- create a clear list of possibilities.

Vocabulary

list
possibilities
systematically

Necessary prior knowledge

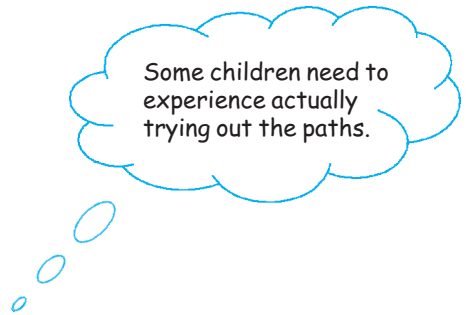
Directions: right, left

  Ref: DfES 0545-2004 G

Main teaching activity

If you have an interactive whiteboard you could draw the routes on the maze.

Show the children the OHT of the problem and explain that this is the maze which Maisie is going to explore. She is trying to find the way out. She must always go forward, never back.



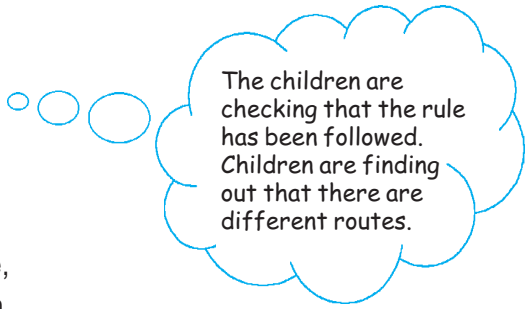
Take the children outside and show them one maze (previously drawn in chalk on the playground). Show them the start and the way out. Ask the children to talk to a partner about which way Maisie could go, to get to the way out.

Ask a child to walk on the paths, keeping to the rule of always moving forward, and find the way out. Using other identical mazes drawn on the playground, let the children have a go at being Maisie.

Drawing together

Come back inside.
Ask a child to draw, on the projected image, the route he or she decided on.

Q. Was she always going forwards?
Agree that she was, and that this is a rule in this problem.
Q. Are there any other ways she could get to the way out?
Give the children copies of the Activity Sheet and ask them in pairs to find another route and draw it on their maze. Say that although there are eight mazes, this doesn't mean there are eight different routes through the maze, but these allow them to experiment and not have to draw all their routes on the same maze.



  Ref: DfES 0545-2004 G

Ask each pair to find other pairs of children and look at their routes.

Q. How many different routes are there?
Gather some answers and record these on the OHT.

Do all the children see that there are other ways?

Q. How can we record our routes?
Discuss that we could draw the routes on the maze and demonstrate that this could be hard to see on one sheet.

Some children might be unable to use this recording but could record their routes on the separate mazes on the Activity Sheet.

Some may need to find the routes by using the playground mazes supported by a TA if available.

Discussing how we know whether we have found all the possibilities can lead children to understand the need for recording in an efficient way.

Q. Would we be able to count the separate routes?
Agree that this would be difficult since they would overlap.

Q. Are there other ways of recording the routes so that someone else could read them and understand them?
Lead the children to using left and right.

Demonstrate the first route by recording:
L, R, L, R as you draw it on the maze.

Do some children have difficulty leaving the visual recording and using L, R? Is this because they have trouble with left and right or is it the need to see the route on the maze rather than visualise and record?

More able children who finish quickly could do a 3 × 3 maze.

Recording L, L, R, R may cause a problem since we don't actually turn left on the second left, we **keep** left. This may need explaining.

Ask a child to show their route on the maze and then record it using L and R.
Tell the children that we are going to make a list of the routes. Demonstrate this by asking another child to record another route under the first.
Ask the children to now record all their routes using this method.

While children are engaged in this activity, look for examples that show different ways of working to use as exemplification later in the lesson.

  Ref: DfES 0545-2004 G

Drawing together

When they have found all the routes they can, ask:
Q. How many different routes are there?
Gather answers and record them on the whiteboard.

Take one set of routes and discuss it.

Q. Are any routes repeated? Have you checked?
Establish the importance of checking.

If they don't notice this, ask how many left and rights there are.

Q. What do you notice about the answers?
Ascertain that there are always 2 lefts and 2 rights.

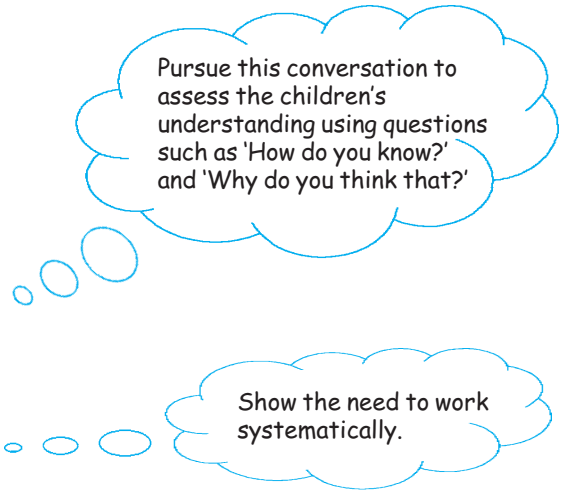
You may need to demonstrate this by drawing an example on the OHT.

Q. Why are there never 3 rights and 1 left?
Ascertain that if there were, Maisie would be going back.

Q. How do we know all the routes are there?
Q. How do we know that we haven't missed any?

Lead the children to understand the need to work and record systematically. Demonstrate by drawing the routes in a haphazard way that we can miss some.

Q. Where could we start so that we work systematically?

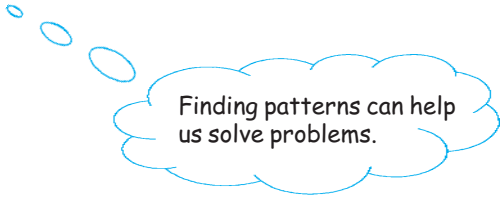


  Ref: DfES 0545-2004 G

If there is no response ask them: Which way does she go? She either goes left or right.

Children who see the symmetry could be asked 'How do you know that if there are 3 routes on the left there will be 3 on the right?'

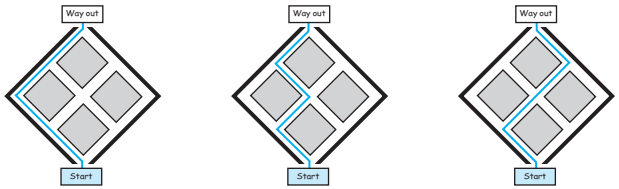
Agree that when we start from the bottom of the maze we can either go left or right and so we could start by going left and finding those routes. We could then go right and find those routes. Some children may see the symmetry and realise that there will be the same number.



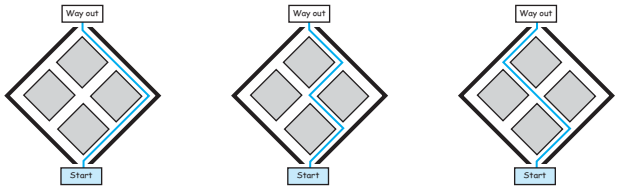
Ask the children to find all the routes systematically and record them systematically.

Drawing together

Q. How many routes are there, starting by going left?
Ascertain that there are 3.



Q. How many routes are there starting by going right?
Ascertain that there are 3.



Ask the children who went back to the picture to think about whether they could have used their list.

Draw out the need for a clear list so that we can count the number of routes.

Plenary

Ask the children to work in pairs to answer:

Q. How many routes would there be if she always had to change direction and couldn't go left, left, or right, right?

Encourage them to look at their lists and discuss whether these can help.

Look at one list and find the possibilities. Lead them to understand that we can use our recorded answers to answer other questions.

Discuss the important points:

- recording systematically;
- checking that we have not repeated any answers;
- checking that we haven't missed answers.

'What if?' questions provide useful extensions to problems.

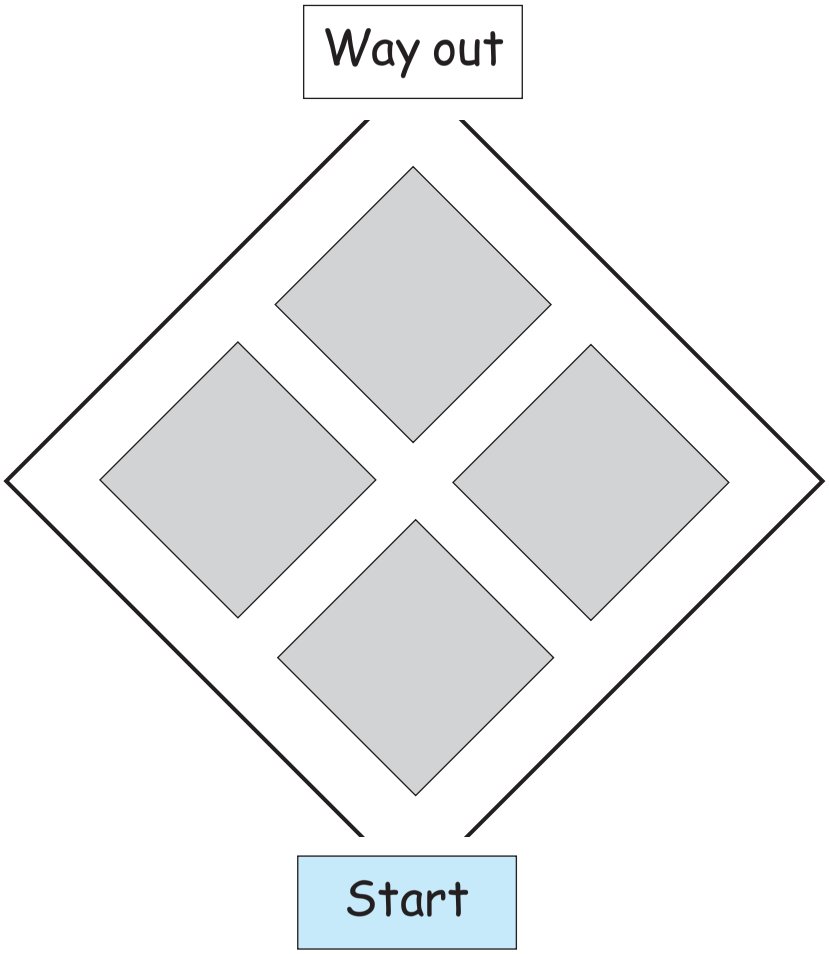
Note whether the children use their recording to find the answer or go back to the picture.

We can use our list to answer other questions.

  Ref: DfES 0545-2004 G

Maisie and the maze

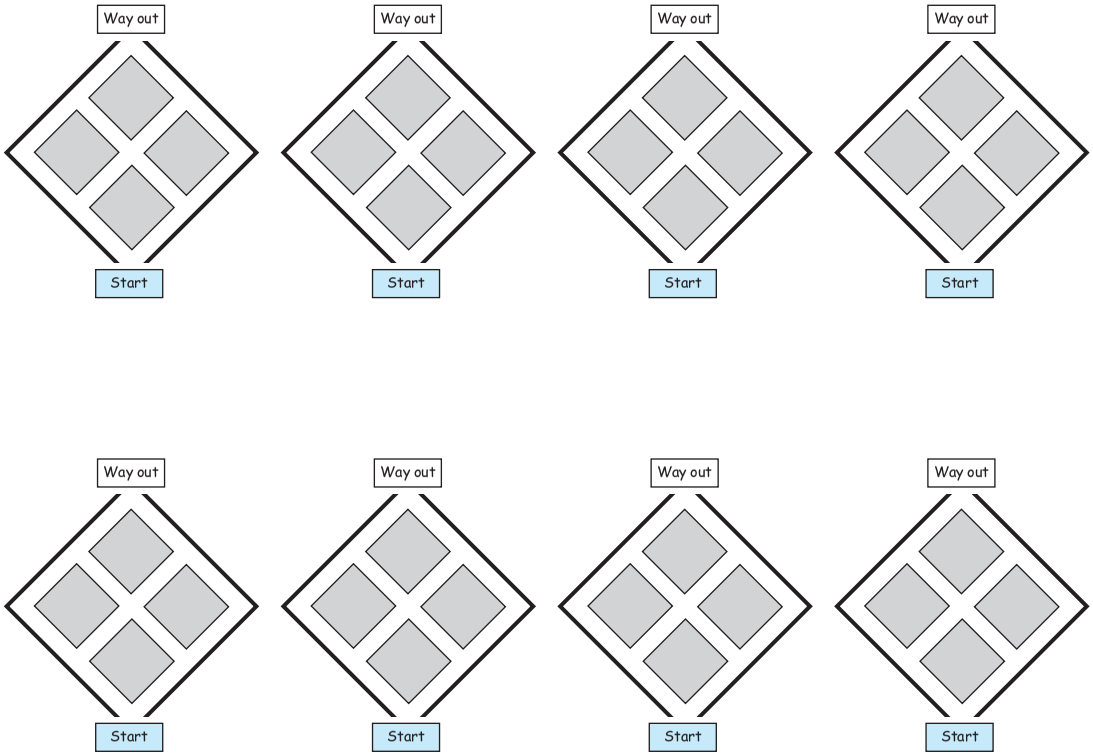
Maisie explored the maze.
She always went forward.



How many different ways are there for Maisie to go from the start to the way out?

Maisie and the maze

Maisie explored the maze.
She always went forward.



How many different ways are there for Maisie to go from the start to the way out?

Finding all possibilities Year 2 Lesson 2

Line of symmetry

You need:
some square paper,
a red pen, a green pen and a blue pen.

Gopal had six squares: two red, two green, two blue.
He put them in a line.
The squares made a symmetrical pattern.

red	blue	green	green	blue	red

Arrange six squares in a line.
Make two squares red, two green and two blue.
Make the line of squares symmetrical.

How many different lines can you make like this?

Resources

- OHP counters, preferably square
- OHT of the problem, from page 28 of *Mathematical challenges for able pupils in Key Stages 1 and 2* (ref. DfEE 0083/2000; NNS publication)
- Coloured squares – some red, some blue and some green
- Squared paper
- Large mirror
- Small mirrors

Objectives

- Solve mathematical problems or puzzles, recognise simple patterns and relationships, generalise and predict. Suggest extensions by asking 'What if…?' or 'What could I try next?'
- Organise the recording of the possibilities, e.g. in an ordered list or table
- Know when all possibilities are found
- Check for repeats of possibilities
- Have a system for finding all possibilities
- Use a list or table to find the answers to other questions

By the end of the lesson, children will be able to:

- find and record all possibilities for a problem in a list systematically.

Vocabulary

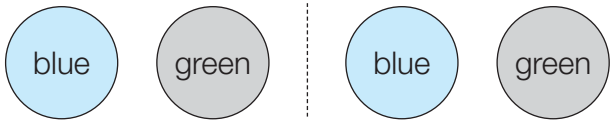
list
possibilities
systematically

Necessary prior knowledge

Symmetry

Main teaching activity

Using OHP counters show a line:



It is key to this problem that children understand symmetry in this context.

Draw a vertical line between the two middle counters.
Ask the children to discuss with a partner whether the counters are symmetrical about this line.
Gather some answers.
Agree that it is not.
Q. Why not?

If you have an interactive whiteboard you could draw counters on it and manipulate them for this activity.

Demonstrate with a mirror that it is not.
Q. How could we make it symmetrical?
Ask a child to move the counters to make it symmetrical.

Establishing the rules shows whether the children have understood the problem.

Show the children the OHT of the problem. Read it together.
Invite children to underline or highlight important words and numbers.
Q. What did Gopal have?
Ascertain that he had 2 red, 2 green and 2 blue squares.
Q. What did he have to do?
Agree that he had to put them in a line and the line had to be symmetrical as we did on the OHP with the counters.

Some children will need to use the mirrors to check the symmetry.

If children can't think of how to start, show them the squares and ask whether they can make a line which is symmetrical. Can they then make another line?

Ask the children to work in pairs to solve the problem. Have coloured squares available, squared paper and mirrors.

  Ref: DfES 0545-2004 G

Drawing together

Q. How did we record our answers yesterday? What was important?
Remind children of how we thought of a quicker and more efficient way of recording using left and right and that we recorded systematically.

Q. Why did we do this?
Agree that it helped us to find all the possibilities and we were less likely to miss any.

Ask the children to remember this when they are doing today's problem and, in pairs, to carry on solving the problem.

Drawing together

Ask some children to describe how they solved the problem.

Q. If the line has to be symmetrical how many of each colour will be in half of the line?
Agree that there will be one of each colour in half of the line and the rest in the other half of the line.

Q. If we start with red, how many different lines will there be?
Agree that there will be two.
The first could be:
red, green, blue.

The problem is left very open and children are not directed to one method of solving the problem. This is a progression from day 1 where the approach was more structured at the start.

How did the children make a start? Note the different ways. Did some children use aspects of yesterday's work?

While children are engaged in this activity, look for examples that show different ways of working to use as exemplification in the plenary.

Which children made more progress with this? How had they started it? Encourage children to think about the different methods used. Did some children realise that their method was not as efficient as others?

  Ref: DfES 0545-2004 G

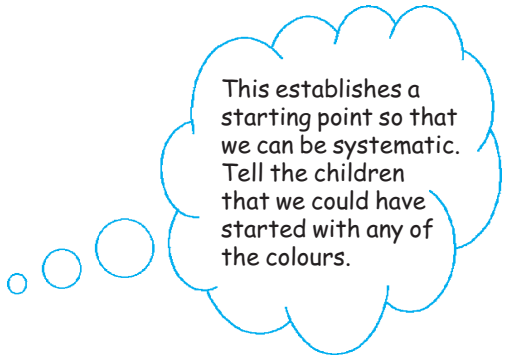
Q. Where do the other 3 counters go?
Ask a child to put them on the OHP:
red, green, blue, blue, green, red
Ask a child to check it with the mirror.

Q. How could we move the counters to get another way?
Agree that the green and blue counters could be swapped. Help a child to do this on the OHP:
red, blue, green, green, blue, red
You could ask a child to check it with the mirror.

Use a child's recording to show to the rest of the class.
Remind them that we are making a list of each possibility:
red, green, blue, blue, green, red
red, blue, green, green, blue, red

Q. If we start with green, how many different lines will there be?
Ask children to discuss this with a partner. Ask them to write down the possibilities.

Collect answers.
Agree that there will be 2 and model recording as was done with red as the first counter:
green, red, blue, blue, red, green
green, blue, red, red, blue, green



  Ref: DfES 0545-2004 G

Q. If we start with blue, how many different lines will there be?
Ask the children to find the possibilities with their partner.
Agree that there will be two possibilities.

Draw out that there were 2 starting with red and then with green so there should be 2 starting with blue:
blue, green, red, red, green, blue, and blue, red, green, green, red, blue.
Show this on the OHP.

Add the last two lines to the list.
Ask the children to look at the list and tell their partner what they notice.

Talk about the lists. Draw out the positions of each colour in the lines.

Q. What do we need to check?
Q. Have we got them all?

Do most children realise that there will be 2 possibilities since there were 2 with red at the beginning and 2 with green at the beginning?

Do they see patterns?
Do they see that all the colours are in all positions?

Remind the children of 2 important aspects of finding all possibilities in problem solving: Have we found all possibilities? Have we repeated any?

Do the children think we have? Ask, 'How do you know?' How do they explain their reasons?

  Ref: DfES 0545-2004 G

Plenary

Children's examples from the lesson are needed and there may need to be prepared examples.

Show the different ways of recording.

There may be:
coloured squares placed on the table,
coloured-in squares on squared paper,
a written form red, blue, etc.,
a shortened written form, e.g. r, b, g

Q. Which is most efficient?
Q. Which takes a long time to record?

If the children find it hard to find the missing one, ask questions such as 'Have we got all the lines that start with red?'

Q. What if I gave Gopal 2 yellow squares? How many possibilities would there be?

Show an OHT of all the possibilities with one line covered up.

Q. Which line is covered?
Q. How do you know?

Which children recognise that r, b, g is an efficient method of recording?

It is not expected that children will find the actual answer but talk about how many there might be.

'What if?'questions provide useful extensions to problems

Do children recognise the patterns and can they identify the missing one?

Which children can explain how they know?

  Ref: DfES 0545-2004 G

Line of symmetry

You need:
some square paper,
a red pen, a green pen and a blue pen.

Gopal had six squares: two red, two green, two blue.
He put them in a line.
The squares made a symmetrical pattern.

red	blue	green	green	blue	red

Arrange six squares in a line.
Make two squares red, two green and two blue.
Make the line of squares symmetrical.

How many different lines can you make like this?

(from page 28 of *Mathematical challenges for able pupils in Key Stages 1 and 2.* Ref. DfEE 0083/2000; NNS publication)

Finding all possibilities Year 3 Lesson 1

Fireworks

Emma had some fireworks.
Some made 3 stars.
Some made 4 stars.

Altogether Emma's fireworks made 19 stars.
How many of them made 3 stars?
Find two different answers.

What if Emma's fireworks made 25 stars?
Find two different answers.

Resources

- OHT of the Fireworks problem, from page 33 of *Mathematical challenges for able pupils in Key Stages 1 and 2* (ref. DfEE 0083/2000; NNS publication)
- Number lines
- Whiteboards

Objectives

- Solve mathematical problems or puzzles, recognise simple patterns and relationships, generalise and predict. Suggest extensions by asking 'What if…?'
- Know when all possible answers have been found
- Organise the recording of possibilities, e.g. in an ordered list or table

By the end of the lesson, children will be able to:

- prove that they have found all possible answers to a problem by generating a list;
- check that their solution fulfils the criterion.

Vocabulary

odd	continue	equals
even	number pairs	list
multiple	total	rule
times	altogether	

Necessary prior knowledge

Recall of multiples of 3 and 4
Addition facts within 20

Main teaching activity

Explain that 'We are going to solve a problem with more than one answer. We need to find all of the answers. We need to think about how we are going to record our working, to know when we have found all of our answers'.
Ask the children to list the first six multiples in the 5× table on the left of their whiteboards. Ask them to circle the largest even number and the smallest even number.
Now ask the children to list the first ten multiples in the 3× table on the right of their whiteboards.
Ask the children to circle all even multiples in both lists above 20 and share their responses with a partner.
Next ask the children to circle all the odd multiples in both lists below 20 and share their responses with a partner.

Q. Which of these odd multiples below 20 is a two-digit number?
Q. What can you tell me about this number?

Establish that it is odd, a multiple of 3 and of 5, below 20 and a two-digit number and that these are the rules that this number fits.

Asking the children to list and circle a multiple or multiples that fulfil a given criterion will encourage and practise the skills that they will need to apply later.

Sharing responses will act as a check for any pupils who have made any omissions in their multiples and encourage dialogue.

The use of manipulatives such as cards/fireworks with 3 stars, 4 stars, number lines, 20 beadstrings, may help some children access the problem.

Show the OHT of the Fireworks problem.
Set a context for the problem. 'Emma's brother is 19 today and she wants to make a firework display with that number of stars, to celebrate.'

Q. Can you describe the problem to a partner in your own words?
Q. What do you already know? How many different sorts of fireworks has she got? How many stars does Emma want in her display?
Q. What have we got to find out?

Ask the children to talk to their partner about how they might start to work out the problem.

Q. Where are you going to start?

Q. How many 3-star fireworks could Emma set off?

Q. How far do we need to go?

Establish that they don't need to set off more than six 3-star fireworks as this would produce 18 stars, and so one more 3-star firework would produce too many stars.

Ask the children to work in pairs to solve the problem.

Give the children just a few minutes for this.

At this point allow children free choice of recording.

If some children are unsure at this point and are working randomly, suggest starting by listing multiples of 3 to find the numbers of stars.

Drawing together

Make a note of the children using a systematic approach and the range and efficiency of recording being used.

Some children may solve the problem quickly through trial and improvement. If so, ask 'What if Emma's sister is having her 25th birthday soon. How many 3-star and 4-star fireworks might she use for her display?' Without telling them that there are two solutions ask them to convince you that they have found them all.

Some children working confidently within 20 may quickly see the pairs of multiples that total 19 and these children also need to prove that these are the only answers through systematic recording.

Q. How did you find the answer?

Q. Does your recording help you? How?

Q. Could someone else understand the way you have worked it out?

Discuss the different strategies and methods of recording.

Remind the children that they will need to know when they have found all possible solutions.

Pursue this conversation to assess the children's understanding using questions such as 'How do you know?' and 'Why do you think that?'

Use a child's example that shows systematic listing and ask that child to explain their thinking.

Draw out that listing allows us to check and convince others that we have found all of the answers.

On the board present the two lists together side by side.

Establish that they need to look for a pair of numbers from two lists, one from each, with a total of 19.

Ask for children's responses to this.

Establish that 5 (3-star fireworks) and 1 (4-star firework) = 19

1 (3-star firework) and 4 (4-star fireworks) = 19

The expected outcomes are more concerned with using lists to prove that they have found all of the solutions to a problem, rather than solving the problem only through listing.

Q. Have we solved our problem?
Q. What did Emma want?
Draw out from the children the criteria.
Re-read the problem, checking against all of the criteria.

Ask the children to solve the second problem on the OHT, i.e. to find the number of 3-star fireworks and 4-star fireworks if Emma's fireworks made 25 stars.

Q. What can we do to help?
Draw out that they can extend their lists and look for pairs with a total of 20. Ask them to do this, and then to pose their own question about Emma's fireworks. They should work in pairs to answer each other's questions.

'What if?' questions provide useful extensions to problems.

Plenary

Collect several of the children's own questions and discuss them together.

Q. If Emma made a display with two 3-star and three 4-star fireworks, how many stars would she have?
Q. What if she lit five 3-star and two 4-star fireworks?
Q. What if it was her 21st birthday?
Encourage children to decide if they need to extend their lists and explain their reasoning. Establish that three 3-star fireworks and three 4-star fireworks, or seven 3-star fireworks, would give 21 stars.

Ask children questions that will encourage them to use their lists of multiples to find pairs that satisfy the criteria.

  Ref: DfES 0545-2004 G

Fireworks

Emma had some fireworks.
Some made 3 stars.
Some made 4 stars.

Altogether Emma's fireworks made 19 stars.
How many of them made 3 stars?
Find two different answers.

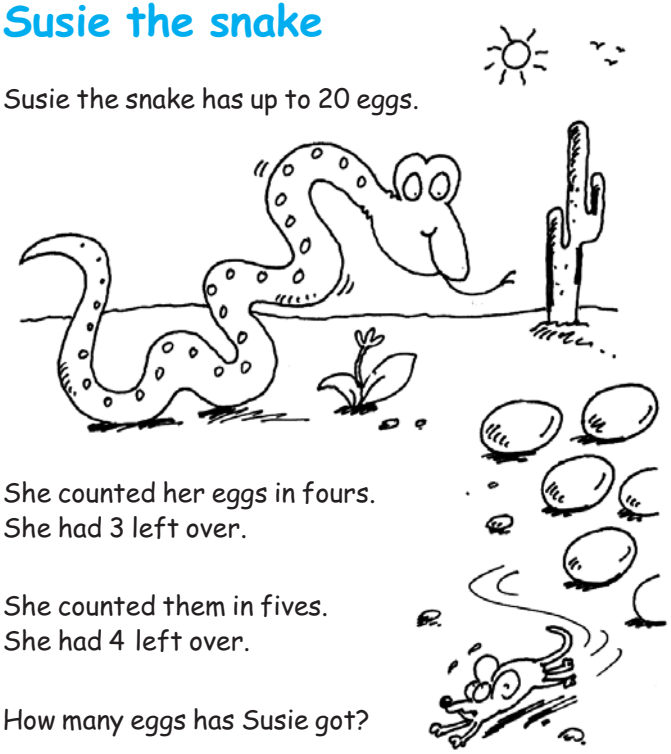
What if Emma's fireworks made 25 stars?

(from page 33 of *Mathematical challenges for able pupils in Key Stages 1 and 2.* Ref. DfEE 0083/2000; NNS publication)

Finding all possibilities Year 3 Lesson 2

Susie the snake

Susie the snake has up to 20 eggs.



She counted her eggs in fours.
She had 3 left over.

She counted them in fives.
She had 4 left over.

How many eggs has Susie got?

Resources

- OHT of the problem 'Susie the snake' from page 30 of *Mathematical challenges for able pupils in Key Stages 1 and 2* (ref. DfEE 0083/2000; NNS publication)
- Bag
- Snake puppet
- Counters/card eggs

Objectives

- Solve mathematical problems or puzzles, recognise simple patterns and relationships, generalise and predict. Suggest extensions by asking 'What if…?'
- Know when all possibilities have been found
- Organise the recording of possibilities, e.g. in an ordered list or table

By the end of the lesson, children will be able to:

- find, and prove, that they have found all possible answers to a problem by generating a list and checking that their solution fulfils the criterion.

Vocabulary

lots of
groups of
multiple of
time
total
altogether
equals

Necessary prior knowledge

Recall of multiples of 4, 5 and 10
Know by heart addition facts up to 20
Understand division as grouping
Understand the idea of remainders
Describe an array

Main teaching activity

Show OHT of Susie the snake to the children.
Set a context for the problem, e.g. Susie the snake has laid some eggs but she isn't very good at counting and isn't sure how many she has laid. To try and find out how many eggs she has, she lays them out in different ways.

Some children may say 7, counting 4 and 3 left over, or 9, counting 5 and 4 left over, without realising that there is a range of numbers that satisfy this criterion.

Q. Can you describe the problem to a partner in your own words?
Q. What do we know? What have we got to find out?
Q. What might the number of eggs be? (9–19)
Q. What might the smallest number of eggs be?
Q. What couldn't the number be? (less than 9, 20 or more)
Ask the children to share their responses to the above with a partner, explaining their reasoning.
Q. Why do you think that?

The use of manipulatives – e.g. a snake puppet and a pile of an unknown number of cut-out eggs or large counters – may help children to access the problem.

This will help children to identify what they know about the possible range of numbers. Listen to the types of responses and ask further questions if needed.

At this point establish that we don't know whether there could be one or more answers.
Remind children about the fireworks problem and discuss what they did that helped them to know when they had found all of the possibilities.

Dependent on the children's responses, it may be useful to have ready some OHT examples of children's recording from that lesson to exemplify the points you want to make.

Q. How will the way you solved that problem help you approach this problem?
Draw out the need for systematic recording to keep a check of their solutions and the use of listing to show that they could convince someone else that they had found all of the answers.
Q. Which number will you start with? Why?

Paired work will encourage dialogue, develop reasoning and help children to feel more secure. It will also act as a check for any pupils who have made errors, through peer checking.

Ask the children to work in pairs to solve the problem.

You may want to use a SMART notebook or EASITEACH page with a pile of 'eggs' ready to discuss the groupings and the relationships they're looking for. This may be particularly suitable for a small group of less able children. Encourage them to annotate and verbalise the groupings and decide whether they fulfil Susie's rules. Encourage these children to continue this method of reasoning for themselves.

Some children may need to list the multiples and then add the leftover eggs.

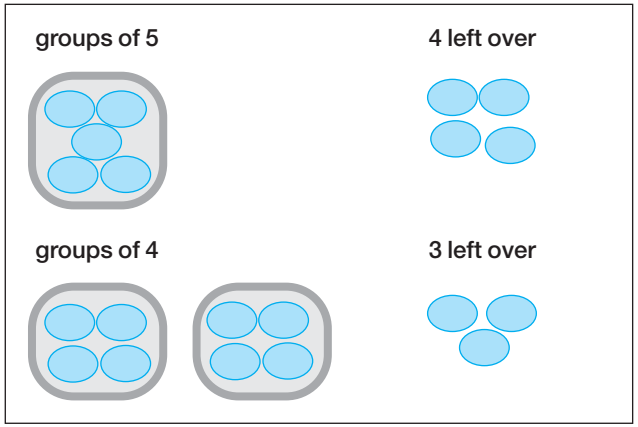
Possible methods of recording:

Groups of 4 + 3 more

4 → 7
8 → 11
12 → 15
16 → 19

groups of 5 + 4 more

5 → 9
10 → 14
15 → 19



Allow enough time for the children to talk about their ideas and methods of recording as they work through the problem.

Note how the level of engagement with the problem compares with Lesson 1, e.g. confidence, speed, systematic recording, checking back with the rules for grouping.

Drawing together

After the children have been working on the task for about 5 minutes, discuss one pair's systematic working or demonstrate this yourself.

Q. Could the answer be 9? Why not?

Try two groups of 5 and 4 more.

Q. Can we arrange these into groups of 4 and 3 more?

Q. Could 14 be the answer?

Ask the children to continue working.

Pursue this conversation to assess the children's understanding, using questions such as 'How do you know?' and 'Why do you think that?'

Plenary

Ask several pairs of children to show the others their recording, and establish that Susie laid 19 eggs.

Q. What was important about your recording that helped you to be sure that you had solved the problem?
Q. When could you use this way of listing again?
Q. What if Susie laid some more eggs? When she arranged them in 10s there were 2 groups of 10 and some left over. She arranged them in 3s and had 2 left over. She arranged them in 4s and still had 2 left over.

Q. Could you think of a different problem similar to Susie the snake's, where you might make lists of multiples and then look for a pair that answers the problem?

'What if?' questions provide useful extensions to problems.

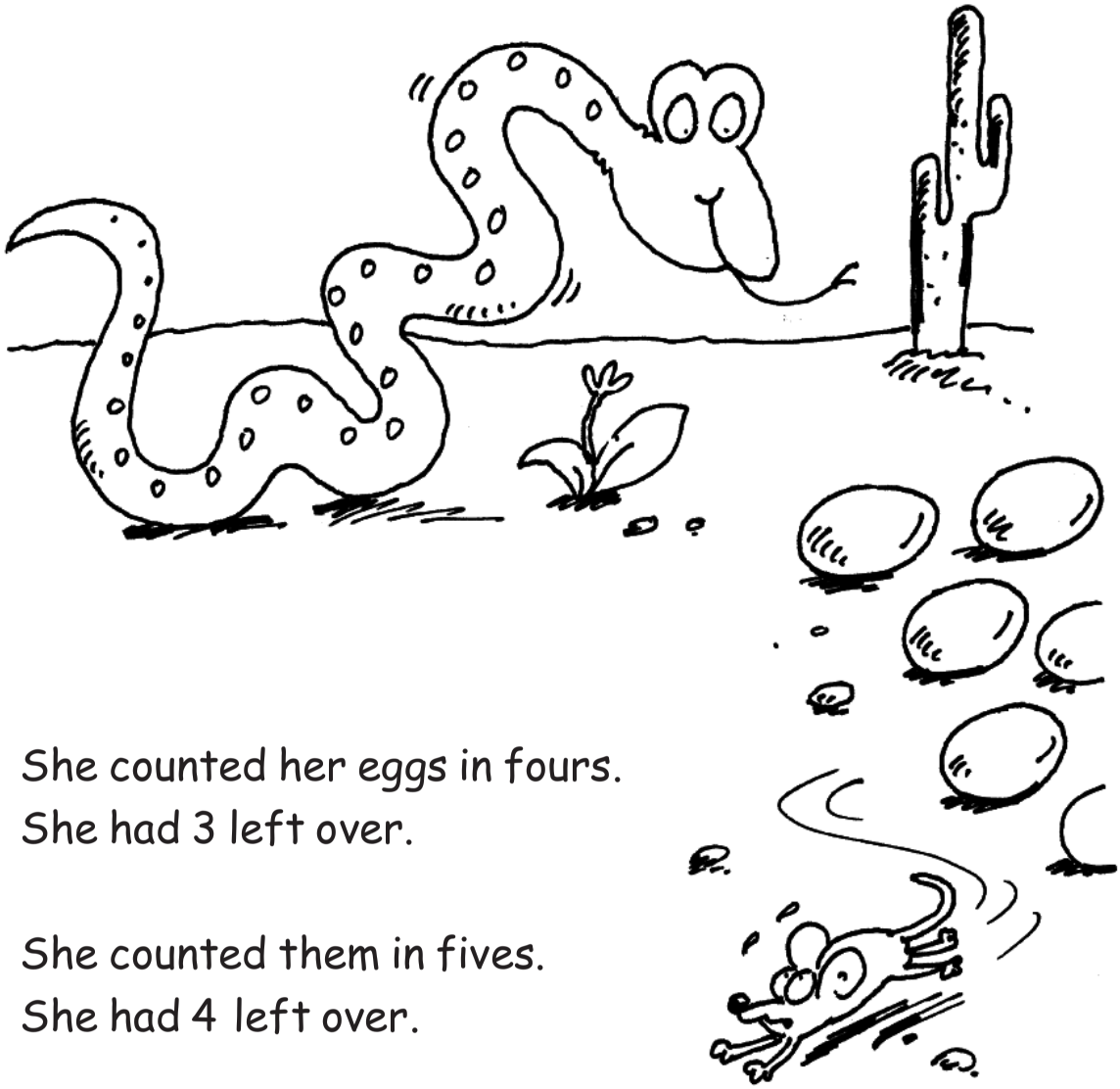
How do they manage this? How appropriate are their examples?

If the children are hesitant, you may need to start them off with a few ideas first.

  Ref: DfES 0545-2004 G

Susie the snake

Susie the snake has up to 20 eggs.



She counted her eggs in fours.
She had 3 left over.

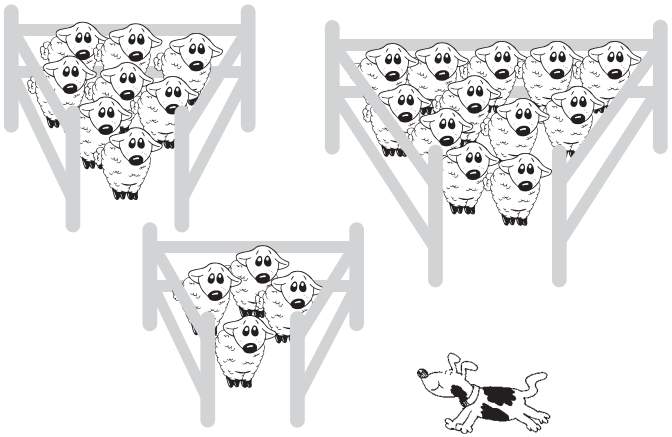
She counted them in fives.
She had 4 left over.

How many eggs has Susie got?

(from page 30 of *Mathematical challenges for able pupils in Key Stages 1 and 2* Ref. DfEE 0083/2000; NNS publication)

Finding all possibilities Year 4 Lesson 1

Sheepdog trials



A farmer and his dog enter a sheepdog trial. In this event his dog must shepherd 24 sheep into three pens of different sizes. Each pen must have a different even number of sheep. The largest pen must have the most sheep and the smallest pen must have the fewest sheep.

How many sheep might the dog try to get in each pen? Find as many different ways as you can.

Resources

- Resource Sheet
- Clip art of 24 sheep to project or 24 magnetic counters and board

Objectives

- Solve mathematical problems or puzzles, recognise and explain patterns and relationships, generalise and predict. Suggest extensions by asking 'What if . . . ?'
- Have a system for finding the possibilities, e.g. start with the smallest number
- Organise the recording of possibilities in a list

By the end of the lesson, children will be able to:

- solve a problem by checking possible solutions against given criteria;
- solve a problem by listing all possible answers;
- list possible answers in a systematic way.

Vocabulary

add
sum
total
odd
even
most/largest/biggest
least/fewest/smallest

Necessary prior knowledge

Recognise odd and even numbers
Add and subtract mentally

Main teaching activity

Explain to the class that they are going to be solving a problem and that there is more than one correct answer to the problem. Explain that in the lesson you want the children to find as many of the possible answers that they can and that together you will be thinking about how they might organise their answers so that they know they have found all the answers.

Give out the Resource Sheet and read the problem with the class.

Q. What is the question we are trying to answer?

Q. What are the important words and numbers?

Invite children to underline or highlight the important words and numbers on the sheet.

Encourage children to visualise the problem:

Q. What are the rules that the farmer must follow?

Record the rules for the class to refer to during the lesson:

- Each pen must have a different number of sheep.
- Each pen must have an even number of sheep.
- The largest pen must always have the largest number of sheep and the smallest pen the smallest number of sheep.

Q. Could a pen have three sheep? Why not?

Q. Could a pen have four sheep?

Q. Could each pen have four sheep? Why not?

Q. What is the smallest number of sheep a pen could have? What is the largest number it could have?

Q. What numbers lie between 1 and 24?

Q. What other numbers of sheep could a pen have?

Ask children to list the numbers on their whiteboards.

If you have an interactive whiteboard and appropriate clip art you could move 24 sheep around to satisfy the rules.

Alternatively magnetic counters could be useful.

Moving two sheep/counters to one side shows the children the 22 counters/sheep left, encouraging them to think these might be split between the remaining pens.

The expected outcomes are more concerned with developing children's problem-solving skills than finding all the solutions. Children need to be able to make a list and check the possible solutions against criteria.

Children should be used to this approach when tackling word problems.

Establishing the rules now will highlight to what extent children have understood the problem. A skill they need to solve the problem will be to ensure that solutions meet these criteria.

These questions should help the children to think about the range of numbers they need to use. Sometimes children write lists that extend beyond the numbers necessary to solve the problem.

  Ref: DfES 0545-2004 G

Share the sheep/counters between the three pens (e.g. 1 in the first, 7 in the next and 16 in the next).

Q. Has the dog followed all the rules? Why/Why not?

Ask the children to help you move the sheep/counters around so that they do satisfy the rules.

Write the solution on the board. Stress that this is only one possibility.

Q. Can you think of another possibility?

Ask children to write this solution on their whiteboards. Ask children to work in pairs to check that each child has followed all the rules.

Paired work promotes discussion and helps children to feel less vulnerable.

If children find all the solutions quickly, ask them to find solutions for a total of 21 sheep with each pen having an odd number of sheep.

Ask the children to work in pairs to find all possible answers.

Drawing together

Stop the class and encourage a child who has been working systematically to share their method with the class.

While children are engaged in this activity, look for examples that show systematic listing to use as exemplification later.

Q. What is the smallest number of sheep that the smallest pen can have? (2)
Q. How many sheep are left now? (22)
Q. What is the largest number of sheep that the biggest pen can have? (20)
Q. Can the largest pen have 22 sheep, or 20 sheep? (No, because that would leave the middle-sized pen with no sheep, or 2 sheep.)
Q. So how many sheep can each pen have? (2, 4, 18)
Q. What other possibilities can you find where the smallest pen only has 2 sheep?

Encourage the children to continue to solve the problem starting with the smallest number.

Drawing together

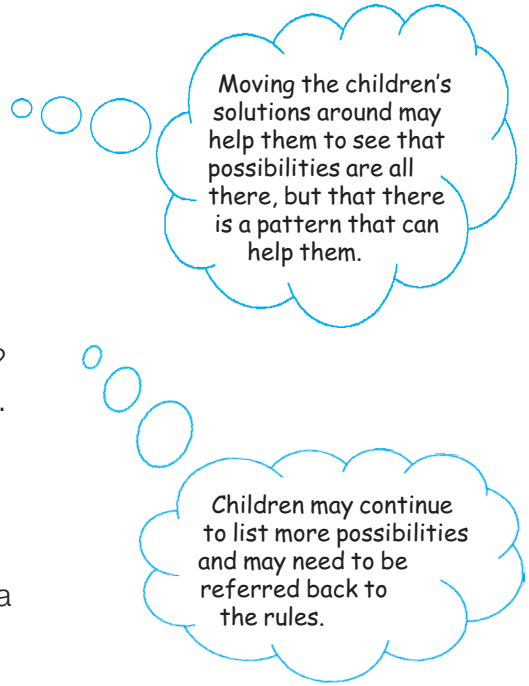
If you have an interactive whiteboard you could record these number sentences on it, and then move them around to show the order as shown here.

Ask the class for all the possibilities when the smallest pen has 2 sheep.
Write these on to pieces of card and stick them onto the whiteboard.
Put these cards into a systematic list:

2 + 4 + 18
2 + 6 + 16
2 + 8 + 14
2 + 10 + 12

Q. Can you see a pattern? How does this help?
Q. Are there any more possibilities where the smallest pen has 2 sheep? How do you know? Explain why.
Q. What's the next smallest number of sheep the smallest pen could have?
Encourage children to continue working systematically finding all possibilities.

You could also copy, paste and drag sheep/counters into a systematic list of groups of sheep. This helps to show that the total remains the same.



Plenary

If you have an interactive whiteboard you could ask children to record their solutions on it, and then move them around to form an ordered list.

Ask the children for all the possibilities that they have found, write these on cards and stick onto board. Ask for a volunteer to rearrange the cards into a systematic list.

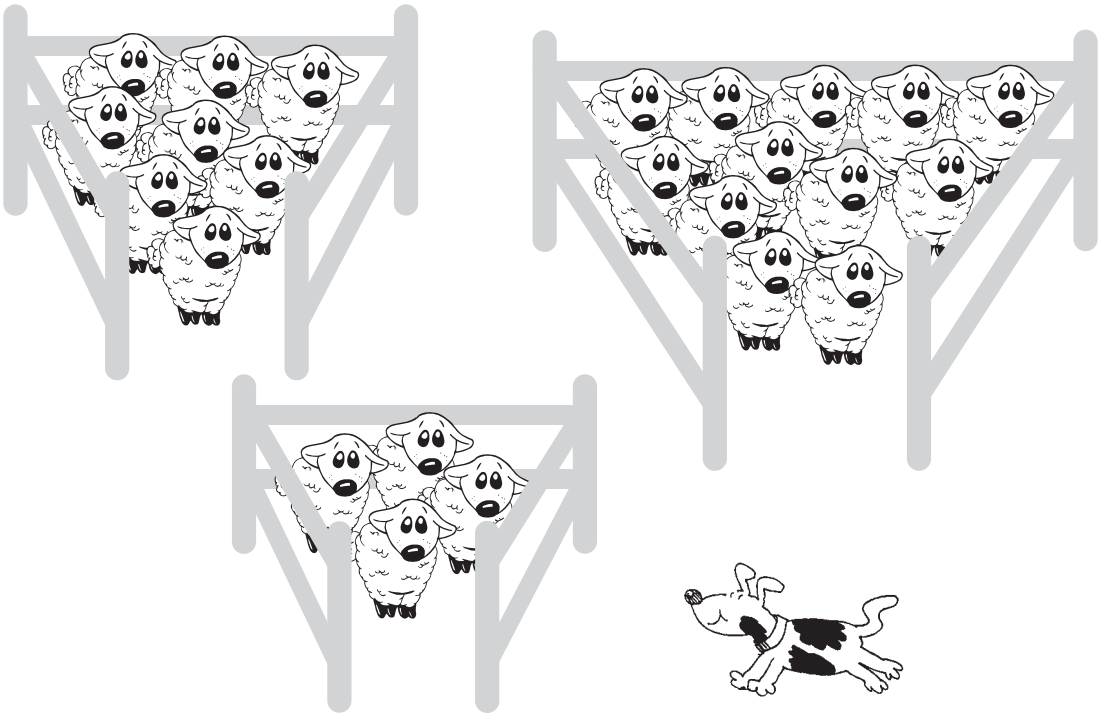
Q. Do all our answers meet the rules?
Q. Have we got all of the possibilities? How do you know?
Q. How many answers are there to our problem? (7)
Q. Which strategies helped us to find all the answers?
Q. We started with the smallest number of sheep. What else could we have done?

Agree that you could have started with the greatest number of sheep in the largest pen.

Q. What would our list have looked like then?
Q. How does working systematically help us?

  Ref: DfES 0545-2004 G

Sheepdog trials



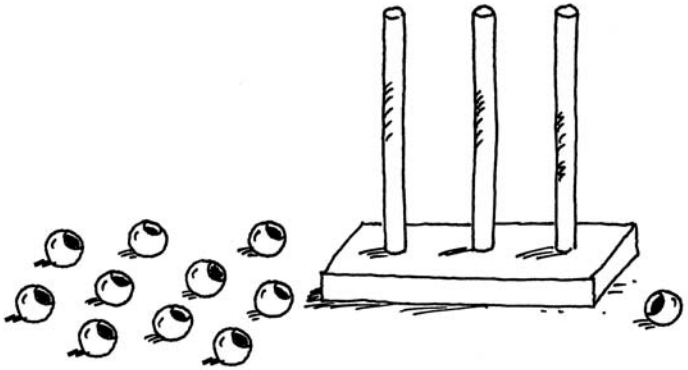
A farmer and his dog enter a sheepdog trial. In this event his dog must shepherd 24 sheep into three pens of different sizes. Each pen must have a different even number of sheep. The largest pen must have the most sheep and the smallest pen must have the fewest sheep.

How many sheep might the dog try to get in each pen? Find as many different ways as you can.

Finding all possibilities — Year 4 — Lesson 2

Three digits

Imagine you have 25 beads.
You have to make a three-digit number on an abacus.
You must use all 25 beads for each number you make.



How many different three-digit numbers can you make?
Write them in order.

Objectives

- Solve mathematical problems or puzzles, recognise and explain patterns and relationships, generalise and predict. Suggest extensions by asking, 'What if . . . ?'
- Have a system for finding the possibilities, e.g. start with the smallest number
- Organise the recording of possibilities in a list

By the end of the lesson, children will be able to:

- solve a problem by checking possible solutions against given criteria;
- solve a problem by listing all possible answers in a systematic way.

Vocabulary

digit sum
add
total
difference

Resources

- Individual whiteboards
- Resource Sheet, from page 13 of *Mathematical challenges for able pupils in Key Stages 1 and 2* (ref. DfEE 0083/2000; NNS publications)

Necessary prior knowledge

Add and subtract mentally
Recall of number facts to 20

Main teaching activity

If you have access to an interactive whiteboard or data projector you could use the *20 cards* ITP.

If children are working systematically, ask them to share their strategies, otherwise model working systematically starting with the smallest possible three-digit number as outlined here.

Write the numbers 126, 512, 421, 460 on the board.

Q. What is a digit sum?

Q. Which of these three-digit numbers has a digit sum of 9?

Q. What is the digit sum of 123?

Q. What other three-digit numbers have a digit sum of 6?

Ask the children to answer on their whiteboards.

Remind the children of how working systematically helped them solve the sheepdog problem.

Q. How did we start the sheepdog problem? What number did we start with?

Draw out that children started with the smallest or largest number and then had to think about what the remaining two numbers needed to be.

Q. What is the smallest possible three-digit number? (100)

Q. Does that total 6?

Q. If we start with a 1 in the hundreds place, what do the other two digits need to total? (5)

Write on the board: 105.

Q. Does this number have a digit sum of 6?

Q. What other numbers could we put in the tens and ones places?

Write the following list on the board:

105
114
123
132
141
150

Q. Can you spot any pattern in how I have recorded these possibilities?

Q. Can you explain it to your partner?

Discuss how we could continue finding digit sums that total 6.

Look for children who are working systematically.

Take just a few of the responses to check children's understanding before modelling a systematic approach.

Which children spot the pattern in the tens and ones columns?

Writing the list in this way should encourage the children to work systematically themselves when tackling the problem.

Paired working promotes discussion.

Q. What might we do next? What number could we put in the hundreds place? Then what do we need to find out about the remaining two digits?

Draw out starting with 2 and thinking of pairs of numbers that total 4.

Explain that we are not going to continue with this problem but will move on to a different problem.

Present the problem to the class, providing a context, for example:

> I have a new car and I would like to have a personalised number plate. The letters will be SLG and I then need to choose a three-digit number. My lucky number is 25 so I would like to choose three digits that have a total of 25. What three digits could I choose?

Q. How might you tackle this problem?
Q. Could working systematically help you?
Q. How will you record all the possibilities?
Q. How will you know that you have all the possible three-digit numbers?

Allow the children to start to solve the problem working independently.

We are not trying to solve this particular problem here. The focus is on modelling and practising problem-solving skills that are needed to tackle the three digits problem.

While children are engaged in this activity look for examples that show systematic jottings to use as exemplification.

One strategy is to choose a starting number and then work out what the remaining two numbers need to total. Some children will use a different strategy: they may have discovered that the digits in 799 have a total of 25 and then simply rearrange the digits to give other possibilities. Look out for this and share with the rest of class.

Some children may find it useful to have digit cards (children will need more than one set to solve the problem).

Drawing together

Q. Has anyone found any three-digit numbers where the digits add up to 25 that have a 1 in the hundreds place? Why not?

Establish that with a 1 in the hundreds place, the remaining two digits would need to total 24 and that it is not possible to have two single digits that total 24.

Q. What other numbers can you not have in the hundreds place?
Q. Why not?

Ask the children to continue, to find all possibilities.

If children quickly find all six solutions set a new challenge:

How many four-digit numbers can you find where the digits add up to 25 but have a 9 in the thousands place? This generates far more possibilities and should encourage the children to work and record in a systematic way.

Plenary

Pursue this conversation to assess the children's understanding using questions such as 'How do you know?' and 'Why do you think that?'

Q. What could the numbers on my new number plate be?
Q. How do you know that you have all the possibilities?
Q. What strategies were useful?
Q. If my lucky number was smaller than 25 would I have more or less choice of digits? Why/Why not?
Q. If my number had to have four digits, what number might I choose?

Ask the children to write a possible solution on their whiteboards.

Q. How did you work it out?

Write one solution on the board (1987).

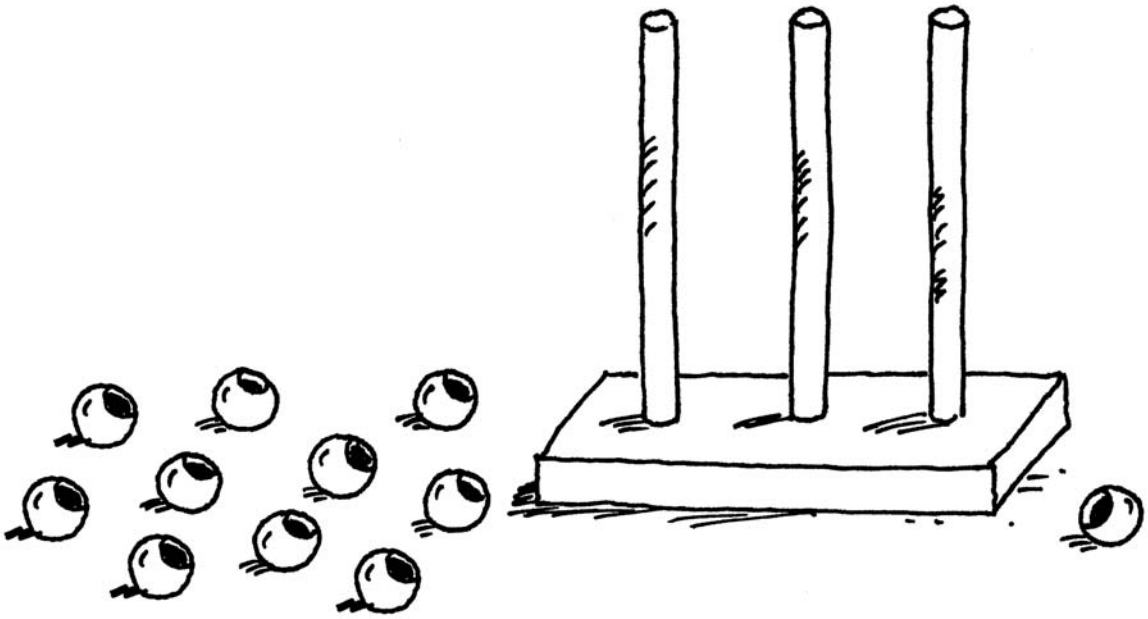
Q. If you know that the digits in 1987 total 25, how could you use this to find other numbers where the digits have a total of 25?

If you have access to an interactive whiteboard, you may want to display some digit cards and ask children to drag the cards around to display different numbers.

This question will highlight several strategies. Some children may rearrange the digits to suggest 7891. Some children will look at the total of the last two digits (15) and suggest numbers such as 1996.

Three digits

Imagine you have 25 beads.
You have to make a three-digit number on an abacus.
You must use all 25 beads for each number you make.



How many different three-digit numbers can you make?
Write them in order.

from page 13 of *Mathematical challenges for able pupils in Key Stages 1 and 2*
(Ref. DfEE 0083/2000; NNS publications)

Finding all possibilities Year 5 Lesson 1

Double scoop ice-creams

How many two-scoop ice-cream cones can we make with two flavours?

Resources

- Crayons
- Coloured counters
- *Number facts* Interactive Teaching Program (ITP)
- SMART notebook

Objectives

- Solve mathematical problems or puzzles, recognise and explain patterns and relationships, generalise and predict. Suggest extensions by asking 'What if…?'
- Know when all possibilities have been found
- Organise the recording of possibilities, e.g. in an ordered list or table

By the end of the lesson, children will be able to:

- find all possibilities by working systematically;
- begin to see how working systematically can help explain reasoning.

Vocabulary

investigate find all
pattern justify
strategy

Necessary prior knowledge

Finding combinations

  Ref: DfES 0545-2004 G

Main teaching activity

Arrange the children so they can work in pairs with rough books or whiteboards.

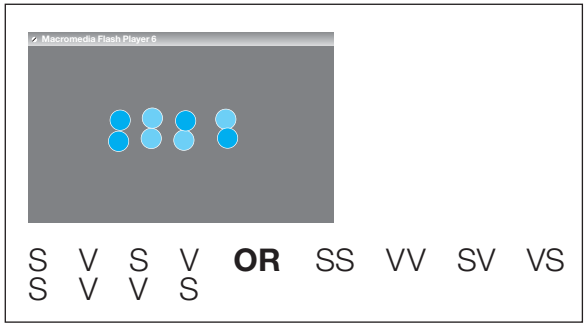
Begin the lesson by posing the problem:

Q. How many different two-scoop ice-cream cones can we make with two flavours?

Ask the children to work in pairs to discuss this and jot down their answers.

Allow the children enough time to think and talk about their own ideas before discussing with the whole class.

Make sure that there are two different coloured counters or crayons available for those children who wish to use them.



It is important to model that there are different ways of recording including the use of colour.

If you have an interactive whiteboard or data projector and screen you could use counters (e.g. from the *Number facts* ITP) to represent the ice-cream scoops. You can then move pairs of scoops around to show the different orders in which children found the solutions.

This whole-class discussion will help you support the less confident and less able children with accessing the problem, by identifying different approaches.

Drawing together

Discuss with the children their findings. Leave a range of different ways of recording to display to the rest of the class.

Discuss with the children their different ways of recording their answers and whether SV and VS are the same or different. Establish that for the purposes of this problem they are different, because the cones are made with scoops one on top of the other, and so there would be four different cones.

'What if?' questions provide useful extensions to problems.

Ask the children to discuss the following questions with their partner:

Q. What if there were 2 flavours but 3 scoops of ice-cream?

Q. Can you predict how many different cones there would be?

Collect some responses and discuss how they might record their findings. Then ask the class to carry on with the investigation.

Look for children who record the solutions systematically so you can show their work to others later.

  Ref: DfES 0545-2004 G

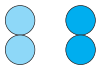
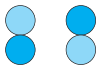
Drawing together

Ask the children to share their solutions in a whole-class discussion, making sure that the following questions are addressed:

Q. How can you be sure that you have found all the different combinations?
Q. How can the way you record your findings help you?
Q. Are there different ways of being systematic with your recording?
Share their different responses and establish that there are 8 different cones possible.

Compare the two problems and how systematic recording is vital if we are to prove that we have found all of the combinations of flavours. If possible use children's work that illustrates this, but if not model the recording yourself.

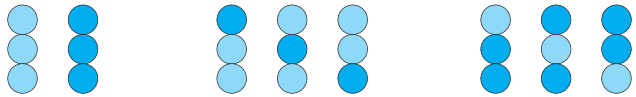
2 scoops:

1 flavour	**1 Strawberry/ 1 Vanilla**	**Total**
V V S S	V S S V	
2	2	**4**
		

  Ref: DfES 0545-2004 G

3 scoops:

1 flavour	1 Strawberry/ 2 Vanilla	2 Strawberry/ 1 Vanilla	Total
V V V S S S	S V V V S V V V S	V S S S V S S S V	
2	3	3	**8**



Ask the children to discuss with their partners:

Q. What patterns and relationships can you see in these recordings?
Q. How is this recording the same or different to your own?
Q. How effective is this recording in ensuring that all combinations of ice-cream cone have been found?

Ask the children:

Q. What if there were 2 flavours and 4 scoops per cone?
Q. Can we use our previous answers to predict the result?

Ask the children to discuss with their partner an effective method of recording and then to work together to find all the possible combinations.

Make sure that you encourage the children to use their recordings to justify their claims that they have found all possibilities.

Discuss with them how identifying a group and then finding all the possibilities within that group is an effective approach. Ensure that they understand that there are different ways of grouping possibilities.

Pursue this conversation to assess the children's understanding using questions such as 'How do you know?' and 'Why do you think that?'

  Ref: DfES 0545-2004 G

Plenary

Establish that there were 16 different cones altogether and that working systematically was necessary to prove that all combinations were identified.

Q. Was it necessary to list all of the combinations of flavours within each group or could it have been possible to work out the total for one group, and then use this information to find the total for another?

We also want the children to realise that being systematic not only ensures that all combinations are identified, but also makes the process of finding all possibilities more efficient.

4 scoops:

The children should spot that there is a pattern in the solutions. If not, ask questions that help them to see the patterns, e.g. comparing the 2nd and 3rd groups.

1 flavour	1 Van/3 Straw	1 Straw/3 Van	2 Van/2 Straw	Total
V S V S V S V S	V S S S S V S S S S V S S S S V	S V V V V S V V V V S V V V V S	V V V S S S V S S S V V S S V V V S S V S V S V	
2	4	4	6	**16**

Once the criteria for a group have been identified these can sometimes be generalised to find the number of combinations in another group.

Through discussion, draw out that 1 Vanilla/3 Strawberry is the reverse of 1 Strawberry/3 Vanilla and so both groups would have the same number of cones.

Q. What if we had 5 scoops; can you predict how many combinations there would be?

Q. Which groups would have the same number of cones in them?

For the final question you do not want the children to make lists but to identify criteria for groups that could be generalised, and so the children begin to see a shortcut to list making.

Finding all possibilities — Year 5 — Lesson 2

Treasure hunt

Jed and Jake are pirates.
Between them they have three precious jewels:
a ruby (R), a diamond (D) and an emerald (E).

Complete the table.
Show what jewels each pirate could have.

Jed	(R)							
Jake	(D) (E)							

Resources

- Crayons
- Counters
- SMART notebook

Objectives

- Solve mathematical problems or puzzles, recognise and explain patterns and relationships, generalise and predict. Suggest extensions by asking 'What if . . . ?'
- Know when all possibilities have been found
- Organise the recording of possibilities, e.g. in an ordered list or table

By the end of the lesson, children will be able to:

- find all possibilities by working systematically;
- begin to see how working systematically can help explain reasoning.

Vocabulary

investigate
pattern
strategy
find all
justify

Necessary prior knowledge

Recording information in a table

Main teaching activity

Give the children the opportunity to talk through their learning.

Display the recording from the previous lesson and ask the children to discuss with their partners what they had learned during that lesson that might help them solve other problems. Take feedback.

Some children may wish to use counters and crayons while other children will wish to represent their recording with letters.

Tell the children that they are now going to use what they learned to solve another problem.

Q. There are two pirates, Jed and Jake. They have an emerald, a diamond and a ruby between them. Which jewels might each pirate have?

Make sure that you stress the point that looking for groupings that can be reversed is an efficient way of using listing as a problem-solving strategy, as they did when finding combinations of ice creams.

Establish that each pirate must have at least one jewel. Ask the children to discuss with their partner how they are going to start the problem.

Q. What recording will you need to use?

Q. How will you decide to group your recordings?

Look for children who have used a table or have found the answers for one pirate and realised that they can use this to find the answers for the other pirate. This problem differs from the ice-cream problem in that they are not being asked to find a number of combinations but to list which jewels each pirate might have, so a table might be a useful way of organising their recording.

Ask the children to find the solution to the problem.

Drawing together

Look for at least two pairs of children who have recorded their work systematically and logically but have made different decisions about the way they devised their groupings or set their recording out. Ask them:

Q. Can you explain the thinking behind your solution? e.g.:

Jed	Jake
R E	D
R D	E
D E	R

Pursue this conversation to assess the children's understanding using questions such as 'How do you know?' and 'Why do you think that?'

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'I knew that one pirate had to have two jewels while the other pirate had one. So once I had found all the solutions for Jed having two and Jake having one, I could swap over the names at the top of the columns and then I would have all the solutions for Jake having two and Jed having one.'

Q. What if the two pirates now had four jewels between them? They have to have at least one each.

Ask the children to discuss with their partner how they are going to tackle this problem.

This is an important teaching point. Once children are able to list systematically, they need to understand that information can be transferred and other lists or combinations can be deduced.

Some children might find it helpful to think about these questions:

Q. Would creating a table be useful?

Q. Would it be helpful to create more than one table?

Ask the children to solve the problem.

While the children are working you should spend time looking for examples of recording to use in the plenary.

Plenary

The focus of this plenary is to help the children evaluate their decisions, strategies and recordings and to communicate their reasoning. The following questions would be useful in helping children to develop and reflect on their learning.

You do not have to ask all these questions in the plenary, but you will need to decide which are appropriate for developing your children's thinking and reasoning.

Q. Can you explain your strategy?

Q. How did you check that you had found all of the solutions?

Q. Can you think of another strategy that might have worked?

Q. Could there be a quicker way of doing this?

Q. Can you make up a similar problem that would be easy to solve using those skills you have been practising?

Q. If you were solving a similar problem what would you do the same? What would you do differently?

Q. What have you learned today?

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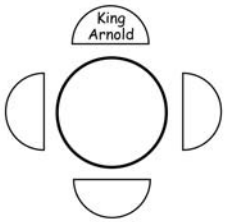
Finding all possibilities — Year 6 — Lesson 1

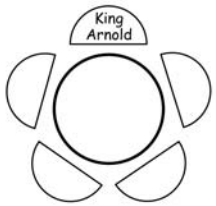
King Arnold

King Arnold sits at a Round Table.



There are 3 empty seats.

In how many different ways can 3 knights sit in them?



What if there are 4 empty seats?

In how many different ways can 4 knights sit in them?

Objectives

- Solve mathematical problems or puzzles, recognise and explain patterns and relationships, generalise and predict. Suggest extensions asking 'What if…?'
- Organise the recording of possibilities, e.g. in an ordered table or list

By the end of the lesson, children will be able to:

- understand strategies to record the solutions to a visual problem in a systematic way.

Vocabulary

systematic
predict
pattern

Necessary prior knowledge

Some experience of systematic recording strategies for problems (see Year 4 and Year 5)

Resources

- Self-adhesive labels or sticky notes
- Large sheets of paper and felt pens
- Resource Sheet of the King Arnold problem, from page 64 of *Mathematical challenges for able pupils in Key Stages 1 and 2.* Ref. DfEE 0083/2000 (NNS publication)

You could use a digital camera to capture possible seating arrangements. If you have an interactive whiteboard or data projector and screen, you could then move these images around to form a list of possibilities.

Main teaching activity

Read the King Arnold problem to the children and explain that the focus of the lesson will be to look at a range of recording strategies and to discuss their strengths and weaknesses.

The focus of this part of the lesson is to try and understand the problem. Role-play should help children to understand and see how they might go about it.

Ask children to work in groups of four and sit on four chairs arranged as in the problem. Give each child a label and ask them to label one member of their group King Arnold and the others Knights A, B and C. Ask them to act out the problem and help each other to move the three knights.

If the children are struggling you may need to ask them to discuss whether pictures, lists or tables might help. Taking digital photographs is another possibility.

After five minutes or so, ask the children to discuss in their groups how they might move the knights systematically.

Q. How might you record it? How have you recorded similar problems previously?

Look out for children who are discussing systematic ways to bring to the whole class discussion. For example, one way might be to keep one knight still and look for ways of rearranging the other knights.

Next ask all the kings to meet together, all Knight As in another group, Knight Bs in another and Knight Cs in a fourth group. Ask them to share how their groups are thinking they might move systematically. They should record the ways on a large sheet of paper.

Ask the children to go back to their original groups and try out a way of finding all the possibilities. Each group should record these on a large piece of paper.

  Ref: DfES 0545-2004 G

Drawing together

Look at each large piece of paper in turn and ask the children from other groups to describe what they think it says.

Q. How do you know you've found all the possibilities?

Q. Did your recording system help you to know how you had them all?

Q. What would happen if there were four knights and four empty seats? Would your recording system still work?

If the children are struggling you could show them the start of one recording system and ask them to complete it.

If some children do this quickly, you could ask them to consider 5 knights and discuss any difficulties this presents.

Ask the children to continue working in fours but to imagine four empty seats and four knights, and to record all the possible seating arrangements. Encourage them to decide first on a recording system.

Q. Are you going to change your recording system? Why/Why not?

The focus here is children evaluating the different recording strategies.

Asking 'What if…?' questions provide useful extensions to problems.

Plenary

Q. What recording strategies have we used?

Q. What were the advantages and disadvantages of each? Discuss the time taken to record the possibilities, the ease with which others could interpret the record and how easy it was to make predictions.

Q. How easy was the strategy to use when the problem extended to four knights?

Q. If you drew a table for three knights, how could/did you adapt this for four knights?

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If children have all drawn pictures of the possibilities show them the following table:

Seat 1	Seat 2	Seat 3
a	b	c
a	c	b
b		

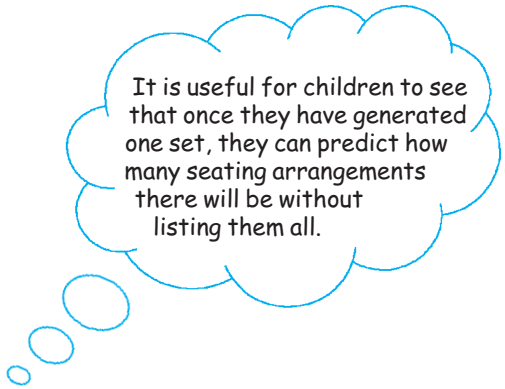
and ask what the next few rows might be.

Seat 1	Seat 2	Seat 3
a	b	c
a	c	b
b	a	c
b	c	a
c	a	b
c	b	a

Draw out that adding an extra column for the 4th seat would be helpful; Knight D could sit here and so there are six possible arrangements with the knight in this place.

Q. What might we do next?
Draw out that we could swap Knight A and D thus creating another set of seating arrangements.

Q. So how many arrangements would there be altogether? Do we need to record them all?
Q. What have we learned?
Q. What is useful for us to remember for another time?



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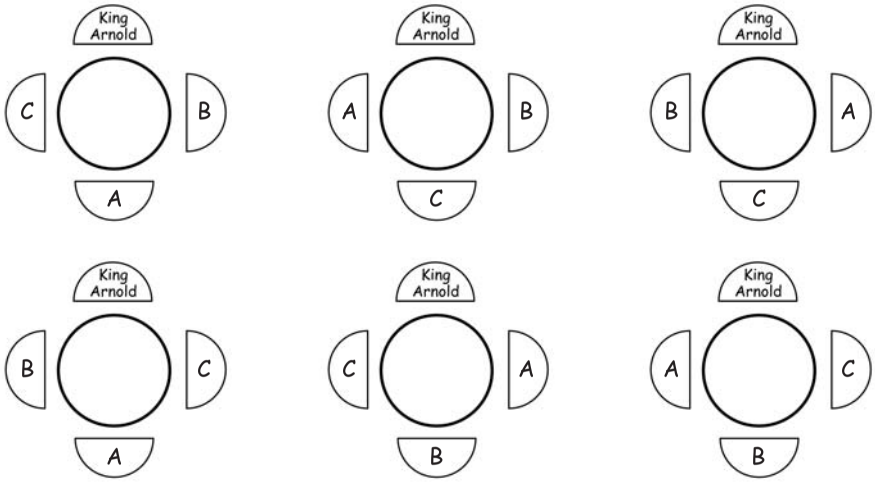
Finding all possibilities Year 6 Lesson 1

Solution (1 of 2)

Possible strategies for recording solutions for King Arnold

Knights lettered A B C Seats numbered 1 2 3

Pictorial Representation



Symbolic Representation

Systematic strategy . . . Move A through each place, move B and C systematically in each of these cases

1A	2B	3C
1A	2C	3B
1B	2A	3C
1C	2A	3B
1B	2C	3A
1C	2B	3A

Systematic strategy . . . Putting each knight in turn opposite King Arnold

1	2	3
A	B	C
A	C	B
B	A	C
B	C	A
C	A	B
C	B	A

Solution (2 of 2)

King Arnold solutions . . .
What happens when King Arnold moves as well, i.e. there are four people on the move?

There are 24 possibilities now.

Build on the strategy used for three knights.
Fix each knight in turn at the top of the table.
Pull out the strong effect of the pattern.

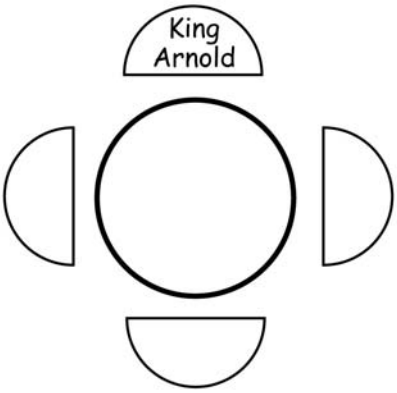
1	2	3	4
A	B	C	D
A	B	D	C
A	C	B	D
A	D	B	C
A	C	D	B
A	D	C	B
B	A	C	D
B	A	D	C
B	C	A	D
B	D	A	C
B	C	D	A
B	D	C	A
C	A	B	D
C	A	D	B
C	B	A	D
C	D	A	B
C	B	D	A
C	D	B	A
D	A	B	C
D	A	C	B
D	B	A	C
D	C	A	B
D	B	C	A
D	C	B	A

King Arnold

King Arnold sits at a Round Table.

There are 3 empty seats.

In how many different ways
can 3 knights sit in them?



What if there are 4 empty seats?

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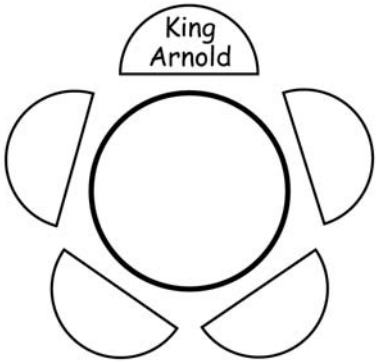


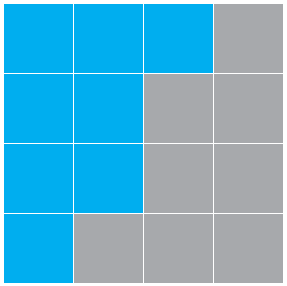
from page 64 of *Mathematical challenges for able pupils in Key Stages 1 and 2*
(Ref. DfEE 0083/2000; NNS publication)

  Ref: DfES 0545-2004 G

Finding all possibilities Year 6 Lesson 2

Four by four

This 4 by 4 grid is divided into two identical shapes. Each shape has the same area.



How many more ways can you find of dividing the grid into two identical shapes by colouring the squares two different colours?

Resources

- Squared paper
- Scissors
- Tracing paper
- Mirrors
- Individual whiteboards
- **Area* Interactive Teaching Program (ITP)
- Software with rotation facility, e.g. SMART

* You will need either an OHT of squares and coloured OHP pens, or an interactive whiteboard or data projector to display the *Area* ITP. Children will need either squared paper or computers with the *Area* ITP loaded on them.

Objectives

- Solve mathematical problems or puzzles, recognise and explain patterns and relationships, generalise and predict. Suggest extensions by asking 'What if . . . ?'
- Understand how to solve problems that involve finding all possibilities
- Organise the recording of possibilities, e.g. in an ordered table or list

By the end of the lesson, children will be able to:

- understand strategies to record the solutions to a visual problem in a systematic way.

Vocabulary

identical
area
shape
square
reflection
rotation
systematic

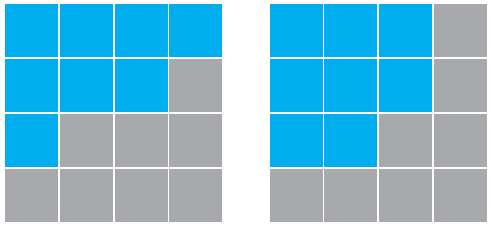
Necessary prior knowledge

Identical, rotation, reflection
Systematic recording strategies for problems (see Year 4 and Year 5)

The *Area* ITP is useful for children to find solutions on.

Main teaching activity

Read the 'Four by four' problem with the children. Check their understanding of the problem. Show two solutions:



Children needing support with the 'identical' feature could cut the shapes out and place them on top of each other to see if they are identical.

Q. Are both of these valid solutions? Why not?
Draw out that although the shapes have the same area, the two parts in the second grid are not identical.

Ask children to work in pairs to produce examples on squared paper.

The focus of this part of the lesson is to allow pupils to get inside the problem and see how it works.

Drawing together

Children working quickly could be challenged further:
Q. Would you be able to do this with a 5×5 square? Explain your answer.
(The grid needs to have even number of rows and columns to avoid half squares.)

Q. How do you know if two parts are identical?
Q. How are you finding different solutions? Do you have a system?
Ensure that children can see that row 1 and row 4 are inverses, i.e. if row 1 has 3 blue squares and 1 grey square then row 4 must have 1 blue square and 3 grey squares.

Ask children to now go and see if they can find all the possible solutions to the problem.

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Drawing together

If children find the solutions quickly they could be encouraged to find other ways of recording, e.g. using a table as in the King Arnold example.

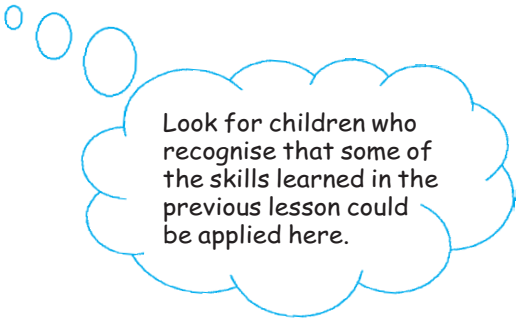
Take different examples of solutions.

Q. How can we organise our answers?

Q. What ways did we have of organising our work in the King Arnold lesson?

Remind pupils of fixing one variable and manipulating others.

Show children a way of being systematic by fixing the colour in one row and varying the answers in next row (see solution).



If you have software with a rotation facility, you could use this to show that some solutions are rotations of each other.

Q. Which of these solutions are repeats?

Agree that rotations and reflections are repeats.

Q. How can we check for repeats?

Encourage children to use tracing paper or mirrors as appropriate to check.

Ask the children to organise their solutions systematically and then discard repeats.

Plenary

Q. How many solutions have you found?

Agree that there are six different solutions, discarding reflections and rotations.

Show the class some of the children's early work.

Q. Where did we start? How organised was our work?

Q. What helped us to become more organised?

Q. What is the value of becoming more organised?

Q. What is useful for us to remember for another time?

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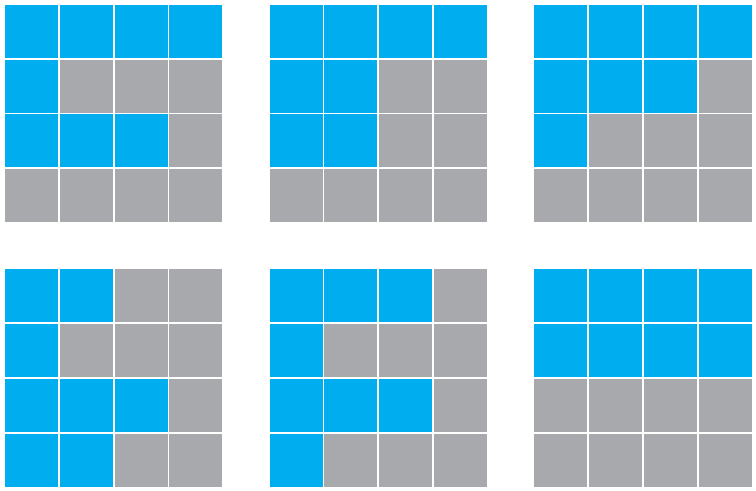
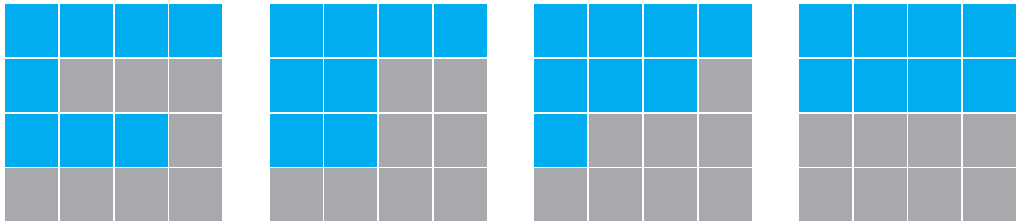
Emphasise that the strategies of being systematic can be applied to visual problems.

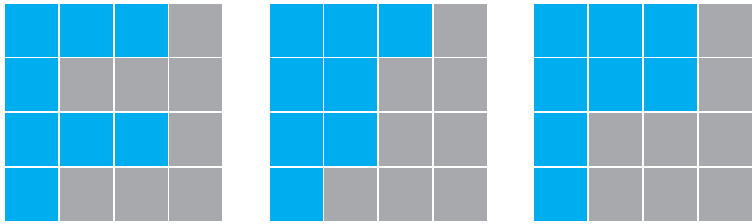
The six solutions are:



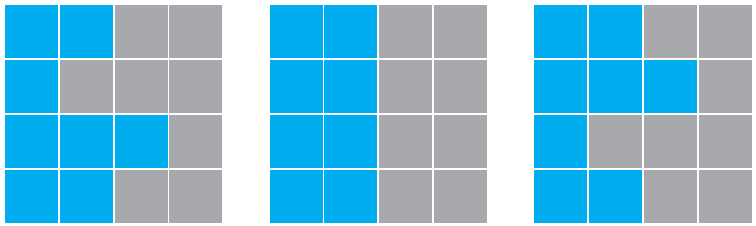
Four blue in first row



Three blue in first row



Two blue in first row



One blue in first row