

Does not understand that 'sets of' or 'groups of' need to be subtracted to solve the problem

Opportunity for: developing mathematical language

Resources

- Cubes
- Bead string
- Wipe-clean number lines
- Calculator

Key vocabulary

- | | |
|-------------|-----------|
| divided by | pair |
| hops of two | sets of |
| subtract | groups of |

Teaching activity

Time 15–20 minutes

Explain to the child that today's activity will help them to divide numbers into equal groups, but that they are going to do it on a number line by drawing hops of the same size.

'We will be taking away equal groups each time because, when we are dividing, we must make equal groups of things.'

? Can you tell me what you think putting things into equal groups means?

Encourage the child to choose anything to show you what they think 'equal groups' means. Build on what the child says.

If the child struggles with equal groups, you can support them by helping them to count to check that each group has the same number of cubes.

? How many cubes has this group got? So how many does this group need? Are all the groups the same size? How many equal groups did you make?

Congratulate the child on any equal groups they made by sharing. Now explain that they are going to make equal groups by taking away along a number line and with a bead string. This is another way of dividing and they are going to be saying '*divided by*' when they use these bead strings and number lines.

Look at a bead string of ten beads with the child. Count them in twos with the child to make sure there are ten.



'Look how we can take away an equal group of two at a time. Subtract two, then another two, then another two.'



? Can you take away another two by moving the beads?

? How many more groups of two can we make?

If the child seems confused, put the beads back at the right-hand end of the string and ask the child to move them two at a time to the other end, saying what they are doing. Support the child through that process.

'There are ten beads. Taking away two means now there are eight left. Take another two so there are six left, and another two and another two, and another two.'

? How many sets of two did we subtract? So how many groups of two are there in ten?

? Can you say a sentence about what we did about subtracting two at a time from ten?

Note the words that the child is able to use and those that need to be taught (see key vocabulary above), for example: 'we hopped back in twos from ten and we did five hops, or we divided ten by two and made five equal groups.' (Put emphasis on *divided by*.)

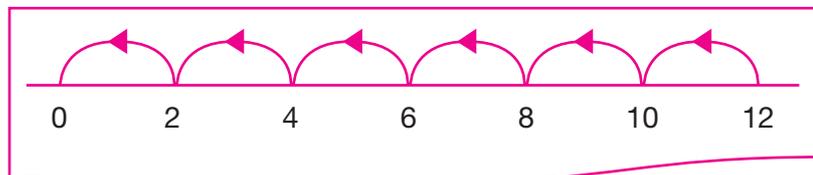
Put two more beads on the bead string, so you are working with twelve.

? How many beads are there now? Do you think we will be able to make equal groups of two? Can you move them two at a time, telling me what you are doing?

Support the child as they talk through what they are doing, helping them to use some of the key vocabulary above, such as:

'We are starting with twelve beads and we are dividing them by two, so we take away a group of two each time. Now we are left with ten beads and take away another two so there are eight left ... and so on.'

Record the process on a number line.



? Can you talk me through what we did?

The child's answer should be similar to the following:

'We started at 12 and we hopped back two to get to 10, another hop of two got us to 8, then another hop of two and we were on 6. Then, taking away another two we were on 4, so we only needed two more hops, a hop to get to 2 and then another and we were on 0 and we stopped there.'

? Can you count how many hops we made? So how many hops back of two are there when you start on 12?

? How do you think we could write this down as a subtraction?

If the child can't cope with this, you could say that you are going to make a very long 'taking away' number sentence.

$$12 - 2 - 2 - 2 - 2 - 2 - 2 - 2 = 0$$

? How many sets of two did we take away?

? So can we say there are six groups of two in twelve?

We can say, twelve divided by two is six. When we divide twelve by two, we end up with six sets of two.

Record this for the child.

$$12 \div 2 = 6$$

? What number did we divide twelve by? (2)

If the child needs more practice, repeat with a bead string of twelve and take away three each time, establishing that there are four groups of three in twelve.

Show the calculator to the child.

? Which two operations are we using today? Can you show me the correct keys?

Stress that when making equal groups you use the division sign, but you can also use the subtraction or take away sign.

? Are you surprised by that? Why?

Spotlight 1

Does not understand that 'sets of' or 'groups of' need to be subtracted to solve the problem

Opportunity for: solving real-life problems

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Snowy day

Time 15–20 minutes

Resources

- Sixteen blue and eighteen black cubes
- Number lines

Key vocabulary

ten, twenty, thirty, etc.	hop back/forward
backwards	subtract
forwards	pair
divided by	sets of
hops of two	groups of

Teaching activity

Explain to the child that today they are going to do some dividing by taking away. They are going to hop back on a number line and this will help them to learn more about dividing by different numbers. They are going to start by dividing by two.

'Pretend your class is away at an activity centre. Pretend that it is a snowy day, so the teacher tells you to count the gloves that have been in the lost property box for a long time, so that people can wear gloves to go out.'

Give the child the sixteen blue cubes and ask them to imagine that the cubes are gloves.

? How many gloves are there? How many children could have two gloves? How could we work that out?

Listen to what the child does and says to solve the problem.

Follow on from what the child says, noting down the words the child uses for division and subtraction.

? What was the picture in your head as you worked that out?

If the child needs support here, suggest that they put the cubes in pairs, like pairs of gloves and then count them.

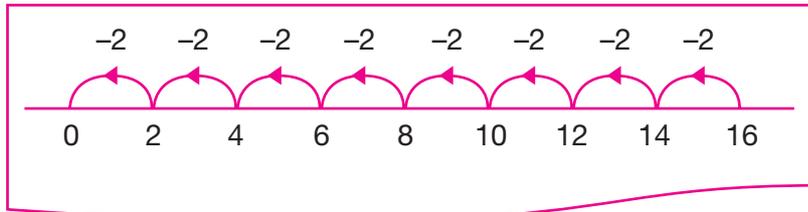
'One pair for one child. Another pair for the next child...'

? How many pairs do we have? So how many children can have gloves?

'Let's do this calculation on a number line. Let's mark 16 here. Now we are going to hop back in twos.'

If the child is unsure, show them how to do a hop back of two, getting to 14.

You might also find it helpful to put the pairs of cubes into one long line, breaking one pair off as the child does the hop back of two each time.



Relate this to a pair of the cubes and also to the repeated subtraction calculation.

$$16 - 2 - 2 - 2 - 2 - 2 - 2 - 2 - 2 = 0$$

? How many are left now? So can you do another hop back in twos?

Keep counting back in twos, taking off the pairs of cubes/gloves as you go.

? How many children can have a pair of gloves?

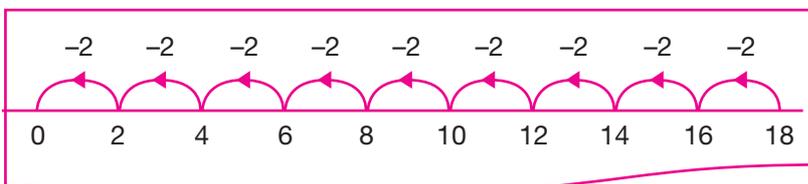
? So what is sixteen divided by two?

For some children it might then be best to choose a smaller number of cubes for the next activity, for example, ten or twelve rather than eighteen.

For children who are ready to move on, you could go back to the snowy day and say that the teacher at the activity centre has found eighteen Wellington boots.

? How many pairs is that? So how many children can have a pair of boots (if they fit!)?

Use another number line and ask the child to mark 18 and hop back in twos.



They might need support to do the related repeated subtraction.

$$18 - 2 - 2 - 2 - 2 - 2 - 2 - 2 - 2 - 2 = 0$$

'We write the eighteen, then we keep counting back in the number we are dividing by.'

? What is the number we are dividing by?

(Keep these repeated subtractions and number line hops to refer to in other sessions.)

Show a formal recording of eighteen divided by two.

$$18 \div 2 = 9$$

'So nine people could each have a pair of Wellington boots.'

Finish by asking the child if they can write the sign we use when we subtract and the sign we use for division.

If the child is not recognising signs, some free or structured play with a calculator can be helpful.

Spotlight 2

Does not understand that 'sets of' or 'groups of' need to be subtracted to solve the problem

Opportunity for: solving real-life problems

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Chocolate bars

Time 15–20 minutes

Resources

- Wipe-clean number line
- Variety of chocolate bars (paper copies of the bars are useful, or double wrap the bars in foil)
- Cubes

Key vocabulary

divided by	pair
hops of two	sets of
subtract	groups of

Teaching activity

Explain that today we are going to do some problems involving dividing chocolate and money, and that we are going to work out the problems on a number line. We have to think carefully about the words that we use to do division calculations, so we are going to write down some of the words used.

? Can you remember doing subtractions on a number line to work out some division calculations?

Show one or more of the number lines with the hops back from a previous session.

? Can you remember what we did with this number line?

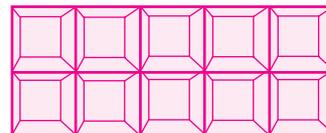
If the child can't remember, talk through what you did. 'We started at 16 and we kept hopping back in twos: 14, 12, 10, 8, 6, 4, 2 and 0.'

? How many hops of two did we make?

Note down any words for subtraction and division that the child uses.

Talk the child through counting forwards in twos on the number line, showing how there are the same number of hops whichever way you go.

Show one of the bars of chocolate to be divided by two.



Talk about the bar and ask the child to count the chunks.

? How many children can have two chunks of chocolate?

Show the division you will be doing.

$$10 \div 2 =$$

'Ten chunks divided by two.'

If the child needs quite a bit of support, and you intend to eat the chocolate, you might find it helpful for the child to make a copy of the bar in cubes.

? Can you break off two chunks for the first child?

Support the child using a range of language as they do the activity (see key vocabulary above).

'If I break off two chunks, that leaves me with eight chunks. Then I can break off another two, leaving me with six chunks ... and so on.'

? So how many children can have two chunks?

You or the child should record what you are doing on a number line. Mark the 10 first. Then you or the child should draw hops of two, counting back from 10 as the chunks (cubes) are broken off in twos.

If the child seems anxious about this, you could start the process and leave one hop almost done so that they can finish it off.



'Starting at ten and breaking off two chunks leaves us with eight. Break off another two and do another hop back of two.'

? How many chunks are left now? Where are we up to on the number line?

Carefully record each stage and talk the child through them, encouraging them to join in to describe the process in their own words. Note down the words they use. (You can use this list to see which words they are not using in the key vocabulary above.)

? So how many chunks of two do we have?

How many children can have two chunks each?

? What is ten divided by two?

? Do you think it would work if we hop forwards rather than backwards?

If the child seems to be understanding, you can make a start on a repeated subtraction and ask them to finish it, letting them talk it through with you as they finish it off.

$$10 - 2 - 2 - 2 - 2 = 0$$

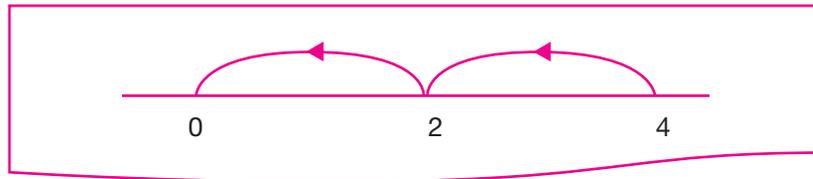
If the child needs more help with counting back on a number line, use a chocolate bar with four, six or eight chunks.



Four divided by two

? How many children could have one chunk?

? How many children could have two chunks?

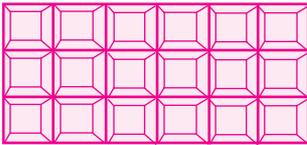


Extend the activity with larger numbers.

Show another chocolate bar with eighteen chunks.

Let the child count the chunks and make a cube copy.

? How many children could have three chunks of chocolate?

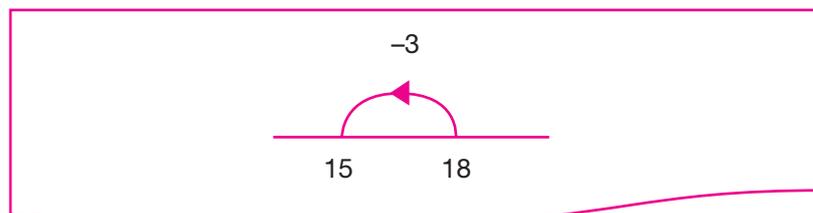


Eighteen divided by three

? Can you draw the number line which we need to work this out? Which number goes at the right-hand end? Can you write the eighteen?

? We are doing eighteen divided by three, so how big do you think the hops back need to be?
(Three)

Encourage the child to do the recording, hopping back in steps of three.



Let the child do as much of the working out as they can, by hopping back in threes and breaking off cubes in threes.

? Can you write down the repeated subtraction that is another way of showing your hops back in threes?

18 - 3 - 3 ...

? How many children could have three chunks of chocolate? So what is the answer to eighteen divided by three?

? Do you see what I mean about having to be careful about the words that we use?
Can you tell me some of those words?

Spotlight 3

Does not understand that 'sets of' or 'groups of' need to be subtracted to solve the problem

Opportunity for: investigating numbers

Is there another way?

Time 10–15 minutes

Resources

- Cubes
- Number cards 0–10 (Resource sheet 1)
- Paper and at least four different coloured pens
- Some of the vocabulary written on card/paper

Key vocabulary

- | | |
|-------------|-----------|
| divided by | pair |
| hops of two | sets of |
| subtract | groups of |

Teaching activity

Explain that today we are going to do some work with twelve cubes and counting backwards and forwards on a number line. We will be using some special division words again today.

Ask the child to count out twelve cubes and make them into one long 'train'.

Now explain that we are going to take away cubes to show twelve divided by two. (Emphasise divided by 2.) That is the number of cubes to be broken off each time, and the size of the hops along the number line.

Ask the child to put a number 2 card onto the right place on the calculation and to read the calculation to you.



? How many cubes are we going to break off each time if we are dividing by two? (Two)

? When you have taken two off, how many are left? (Ten)

Mark 12 on a piece of paper, making the line about the same length as the 'train' of twelve cubes.

? What size of hops are we using if we are dividing by two? (Two)
Can you draw a hop back of two with the green pen?



? How many cubes are left on the cube 'train'? (Ten)

? What number are we down to on the number line? (10)

? What number are we dividing by? (Two)
So how many should you break off this time? (Two)

As each pair of cubes is taken off, record the step of two on the number line and count how many cubes are left on the cube 'train'.

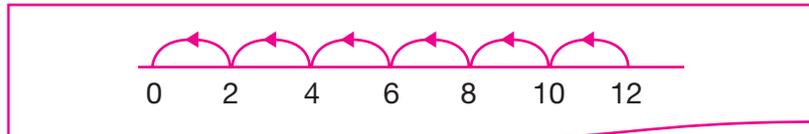
Keep the broken-off pairs of cubes as towers of two.

If the child counts the cubes in the cube 'train' one at a time, encourage them to count back from the last number, so twelve take away two is ten. If the child is struggling with this, jot down two, four, six, eight, ten, twelve on a bit of paper and remind them of counting in twos, forwards and backwards.

Keep repeating: 'We are dividing by two so we break off two each time.'

? How many cubes are left on the cube 'train'?

What number are we down to on the number line?



? How many hops of two did we make when we divided twelve by two? (Six)

? What is the answer to twelve divided by two? (Six)

? Can you say the answer in a number sentence, using the words divided by two?

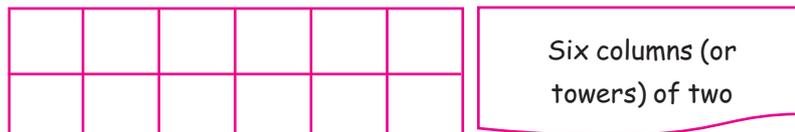
Record for the child:

Twelve divided by two makes six equal groups of two.

? Could you say that number sentence in a different way?

For twelve divided by two you make six equal hops of two.

Ask the child to make the broken-off pairs of cubes (kept as six towers) into an array.



? Can you tell me how many rows and columns there are in this array? How many cubes are there altogether?

Establish that there are six columns and two rows.

? If you shut your eyes, can you see the cubes in your head, arranged in six towers or columns of two?

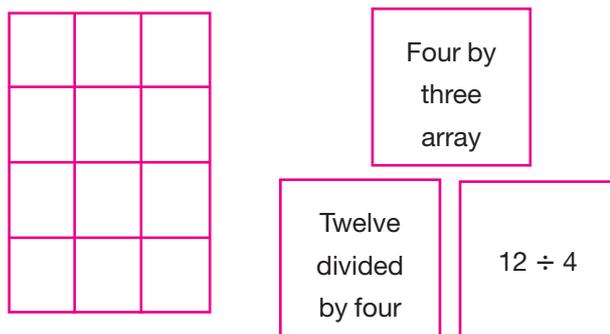
? Can you use those same twelve cubes to make a four by three array?

If the child finds that hard, help them to put together a four by three array. Establish that there are still twelve cubes, but that we have arranged them differently.

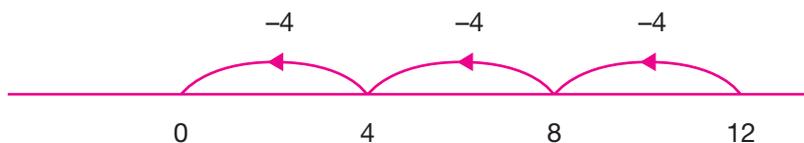
? What size hops do we do when we are dividing by four?

Can you draw those hops in blue?

If the child needs help with that question, lead them towards breaking the array up by taking four off each time.



On the number line they should start at 12 and land on 8, 4 and 0, making three hops of four.



Establish that the number which you are dividing by is both the size of the hop back and the number of cubes you break off each time.

If you have time, you could work with the child to make other possible arrangements of twelve cubes. With a red pen, you could do four hops of three, or twelve hops of one, or one hop of twelve, or two hops of six, or six hops of two.

? Can you think of another way to arrange the twelve cubes as an array or rectangle?

? Which other numbers are like twelve, so that you can make several different sizes of hops going from the number to zero each time?

Spotlight 4

Does not understand that 'sets of' or 'groups of' need to be subtracted to solve the problem

Opportunity for: developing mental images

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Ribbons, puppets and pennies

Time 10–20 minutes

Resources

- Cubes
- Two metres of ribbon
- Metre stick
- String and scissors
- Coins
- Buns or circles of paper
- Finger puppets
- Two sets of number cards 0 to 40 or 50 (Resource sheets 1, 2, 3 and 4)
- Some of the number line drawings from previous activities

Key vocabulary

divided by	pair
hops of two	sets of
subtract	groups of

Teaching activity

Explain that today we are going to do some dividing using a number line and that they can use any of the things on the table to help them.

You could show some of the previous number line drawings.

Write an 'empty' number sentence for the child to use for each problem, putting in number cards in appropriate places.

<input type="text"/>	divided by	<input type="text"/>	makes	<input type="text"/>	equal groups of	<input type="text"/>
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? What can you tell me about dividing using a number line?

Show the child a bit of ribbon that is 40 cm long. Put the ribbon next to the metre stick.

? If we divided this ribbon into 10 cm pieces, how many pieces would we cut it into?

Establish that to divide by ten you would cut it into four pieces.

Ask the child to shut their eyes and try to see a picture in their mind of the ribbon divided into four pieces.

If you think that the child needs to do some folding and cutting, cut paper strips or string and model the activity above. Create examples based on 20 cm or 30 cm if you think more examples would be helpful.

Now tell the child that you are going to ask them some questions and that they should decide how they are going to answer them.

'Some children are writing plays to perform in a puppet theatre. Their teacher has a total of eight finger puppets.'

? How many children can have one finger puppet on each hand?

? How will you work that out?

If the child seems confused, repeat the question. If they still don't know how to work it out, suggest that they get some cubes (ideally ones that will fit onto a finger like a finger puppet does) and also take a pen and paper and draw a number line with eight at the end.

Clarify that they are dividing eight by two.

? When we divide by two, how many do we need to take off each time?

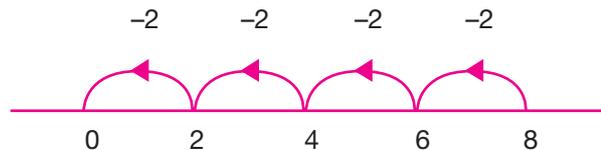
Ask the child to shut their eyes and try to see the eight finger puppets in their head.

? Can you see a number line to eight in your head?

? We are dividing by two, so what does that tell you we need to do? (Make hops of two on the number line and/or put the cubes in pairs.)

If the child needs support to do this, help by making the hops back along the number line, counting back in twos.

Link this to moving two of the cubes (see *Spotlight 3*). 'We have eight cubes, so we need to take away two of them for the first pair of puppets.'



? How many hops of two do we make when we divide eight by two? What is the answer?

Help the child to make a sentence explaining what they have done, using the key vocabulary.

We had eight finger puppets and we divided them by two and made four equal sets of two. So four children can have two finger puppets each.

Select from the five problems below.

They can all be solved by repeated subtraction. For each problem, write the calculation (or fill in the 'empty' number sentence above) and count on or back on a number line.

You can change any of the numbers to suit the child. (The only one with a remainder is for the whole class to solve.) Let them choose freely from mathematics equipment to help them solve the problems.

Each time ask:

? How did you work that out? Was there a different way of doing it? Can you see the number line in your head?

? 1. Sally has sixty pence. How many 10p sherbet dips can she buy?

$60 \div 10 = 6$ 'Sixty divided by ten is six, so Sally can buy six.'

Some children might solve this quickly mentally. Praise them for that, and ask them how they did it.

Many children find it hard to explain their methods, but if they have been having problems with the number line hops, you could draw this one for them to show them what they might have done in their head – but they did it so quickly they couldn't 'see' it! Start at either 0 or 60 and count in tens to make six steps of ten.

? 2. Tim's hens have laid twenty-four eggs. How many boxes of six can he fill?

$24 \div 6 = 4$ 'Twenty-four divided by six is four, so Tim can make four full boxes.'

? 3. There are twenty-six slices of bread in a loaf. How many people can have two slices each?

$26 \div 2 = 13$ 'Twenty-six divided by two is thirteen, so thirteen people can have two slices each.'

? 4. There are thirty-two strawberries in a punnet. How many people can have eight each?

$32 \div 8 = 4$ 'Thirty-two divided by eight is four, so four people can have eight each.'

? 5. Abdul is making five clay bugs. He wants to use drinking straws for legs. Each bug needs six legs. He thinks he can cut straws in half and they will be about the right length. How many straws will he need? (15)

? How did you work that out?

? Was that a hard question? Why?

For question 5, it might be useful to ask children to record in their own way how they worked it out. It is a two-stage question and children might have added six five times, then halved the result. But they might have many different ways of solving it.



? Mrs Bun the baker packs her sticky buns in fours. If she makes forty buns, how many full packs can she make?

? One day she found that she had three buns left over after packing them. How many buns might she have made that day?

? Write a story problem for thirty divided by five. Make a number line hopping picture for your story. Did you use counting back or counting forward? Can you use either?

Spotlight 5: a learning check

Does not understand that 'sets of' or 'groups of' need to be subtracted to solve the problem

Opportunity for: discussing and explaining

Stick break

Time 10–20 minutes

Resources

- Cards with 'divide by two', 'divide by three', 'divide by ten' written on them
- Cube 'trains' of eight, nine, ten, twelve, fourteen, fifteen, sixteen, eighteen, twenty, thirty
- Some extra 'ten trains'
- Sticky tape
- Numbered number line to 30
- 100-square
- At least one other child to play the game

Check: does the child use key vocabulary?

divided by	pair
hops of two	sets of
subtract	groups of

Teaching activity

Explain that today they are going to play a game called **Stick break**. Show them the cube 'trains' and say that they are the 'sticks'. Explain that we will also use the following cards.



The aim of the game is to end up with no cubes at all.

Put the sticks on the table and label each one with the number of cubes. Put the cards face up on the table.

How to play

1. Players take turns to choose a stick and a number card, for example, the twenty stick and the 'divide by ten' card.
2. The players then need to work out how many sticks to break off the long stick so that there will be no cubes left. They must break up the stick and say what they are doing in a number sentence, reading the card they chose. For example: 'Twenty divided by ten makes two equal sets of ten with none left over.'
3. If everyone agrees that they have said their number sentence correctly and have worked out the numbers correctly, they win no cubes – which is a good thing! They need to try not to win any cubes! If they do win cubes they are likely to lose!
4. The stick is put back together again, and the card and stick remain on the table for others to use.
5. If a player chooses a stick and a card where they do get a remainder, they will win ten cubes. For example, if a player chose the eighteen stick and the divide by ten card, they would not be able to make equal sets of ten.

If children find the game hard, support them with counting in twos, threes and tens, with either a number line or 100-square to help.

Prompt children to listen very carefully.

When all the children have had a few turns, ask them to stop and reflect on what they have done.

? Which sticks are good to divide by two?

? Why did you choose the divide by ten card for that stick?

Variation

- Play the game with piles of coins, for example, fifty pence, thirty pence, one pound, eighteen pence, forty-five pence, seventy-five pence, and so on.

? Do we need to make some more 'divide by' cards? Which numbers do we need?

Learning outcomes

By the end of this set of activities, children should be able to:

- tackle related learning tasks with increased motivation and confidence;
- use and understand connected mathematical vocabulary;
- understand and use a range of vocabulary for division;
- recognise that when *divide by* is used, the problem can be solved by repeated subtraction;
- form mental images of arrays and number lines;
- begin to learn some division facts.