

## Interprets division as sharing but not as grouping (repeated subtraction) so is unable to interpret a calculation such as $12 \div \frac{1}{2}$

**Opportunity for: solving real-life problems**

### Resources

- Apples, muffins, chocolate bars, dough
- Plastic fraction parts or *Fractions* (Resource sheet 42)

### Key vocabulary

halves	divided by
quarters	how many halves in ...?
whole ones	divided into halves
fractions	

### Teaching activity

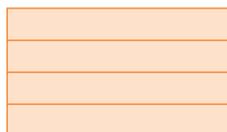
**Time** 15–20 minutes

'We are going to do some dividing by fractions. I've got some apples/muffins/chocolate/dough to help you understand what we mean by dividing by fractions.'

'I'm going to write down some of the words and symbols that we use when we divide by fractions.'

Show a chocolate bar (or paper copy) that could be divided into quarters.

**? How many quarters in a whole one?**



**? How many quarters can you get from three whole ones?**

If the child is unsure about the word 'quarter', you will need to do a variety of practical activities focusing on fractions. Understanding fractional parts needs to be secure before working through this set of activities.

You or the child can draw the three chocolate bars to show the twelve quarters and record words and symbols.

Words and symbols we use to divide by fractions.



How many quarters in three whole ones?

Three divided by a quarter is twelve.

Three divided into quarters is twelve.

$$3 \div \frac{1}{4} = 12$$

**? How many half apples can you get from five whole ones?**

Let the child work this out any way they choose.

If the child can't seem to visualise what is needed, use apples or paper strips to improve accuracy and actually cut the five apples into halves, or fold and cut the paper.

You might want them to draw what they did on the recording chart.

It would be helpful to talk through and draw at least one calculation on a number line.

How many halves in five whole ones?

Five divided into halves is ten halves.  
 Five divided by a half is ten.  
 $5 \div \frac{1}{2} = 10$   
 How many halves in five?

**?** Can you use these six muffins to make up a division sentence with 'divided by half' in it?  
 How did you work that out?

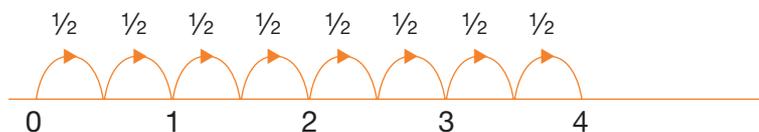
If the child needs more help doing this, let them work practically with further objects, such as dough, paper strips, and so on.

Give more examples and encourage them to make up their own ones, such as 'Fourteen oranges divided by a half means that twenty-eight chimps can have a half each.'

You could do a recording in which they select the right words from your previous recordings.

So on a number line you could draw six divided by a half, showing the twelve half hops that can be made.

**?** Can you make up a 'dividing by' number sentence that will go with this number line picture?



This could be 'Four apples divided by a half makes eight halves.'

**?** What did you learn today about dividing by fractions?

Keep all the recordings of words, symbols and pictures for next time.

## Spotlight 1

Interprets division as sharing but not as grouping (repeated subtraction) so is unable to interpret a calculation such as  $12 \div \frac{1}{2}$

### Opportunity for: developing mathematical vocabulary

#### Dividing words

Time 15–20 minutes

#### Resources

- Apples, muffins, chocolate bars, dough, blunt knife
- Plastic fraction parts or *Fractions* (Resource sheet 42)

#### Key vocabulary

halves	how many halves in...?
quarters	divided into halves
whole ones	half of
fractions	divided into half
divided by	

#### Teaching activity

Explain to the child that today's activity will help them to use mathematical words for dividing by fractions.

First, you will be showing similarities in the division vocabulary when we divide by whole numbers.

#### ? Can you read this number sentence: $10 \div 2 = ?$

Record the words used for grouping – these are used below – comparing them with the words used to divide by fractions.

$10 \div 2 =$

Ten divided by two.

How many twos make ten?

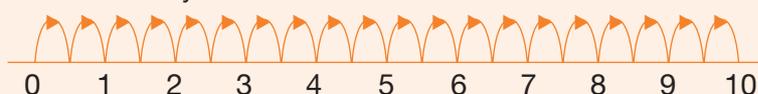
Ten divided into twos.

If the child uses some of the vocabulary of sharing, such as 'divided between' or 'shared between', you need to explain that today you are thinking of 'jumps along a number line kind of dividing' or 'grouping'.

If the child needs more support on this, you might want to draw a number line and model 'how many twos make ten'.



'Five twos make ten. How many halves make ten?'



'Twenty hops of a half make ten, because there are two halves in each whole one.'

Show the recordings from last time when you divided by fractions, for example dividing ten by a half.

We used these words when we divided ten by a half.

$$10 \div \frac{1}{2}$$

Ten divided by a half.

How many halves make ten?

Ten divided into halves.

Put the two sets of dividing words which you have recorded side by side to show the similarities in the words we use.

'When we divide by numbers smaller than one (fractions) we can use the same words as when we divide by a number that is one or more – a whole number, such as two or four or ten. This dividing by a fraction can be worked out on a number line, just like dividing by a whole number.'

? **Can you read me this number sentence:  $8 \div \frac{1}{2}$ ?**

? **Now can you say the sentence in a different way?**

? **What is the answer?** (*Sixteen halves make eight.*)

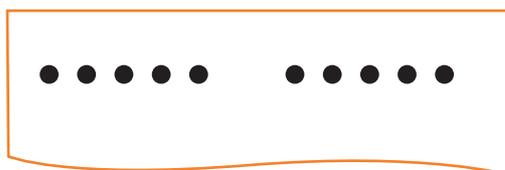
If the child needs more help, return to some of the examples used last time and use different numbers, for example, twelve cakes divided by a half, or ask the child to make up their own examples. You can record for them.

Each time, make sure that the child is using appropriate words.

You could highlight important words with a bright highlighter pen.

'Now we are going to make some pictures in our head for two different number sentences.'

? **What is half of ten?**



? **How many halves are there in ten? How could you draw that?**

Help the child to draw either a number line showing twenty halves, or ten buns divided into halves.

Encourage the child to think very carefully about what these two different sentences mean.

Put the two number sentences together with their pictures, to compare them.

If the child needs help, they could use ten of any suitable resource and show that half of ten is five, then use ten apples and cut them in half to show there are twenty halves in ten.

'We need to be very careful about how we use words, and we need to find a good mental picture to fit with the words.'

Some of the highlighted words could go up on display in the classroom. The recordings will be needed for *Spotlight 2*.

## Spotlight 2

Interprets division as sharing but not as grouping (repeated subtraction) so is unable to interpret a calculation such as  $12 \div \frac{1}{2}$

### Opportunity for: making generalised statements

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### Larger or smaller?

Time 15–20 minutes

#### Resources

- Some earlier recordings
- Calculator

#### Key vocabulary

halves	divided by
quarters	how many halves in...?
whole ones	divided into halves
fractions	

#### Teaching activity

'We are going to look at what happens to numbers when you divide by a whole number, and how this is different from dividing by a fraction.'

**? If we divide ten by two, is that dividing ten by a whole number or by a fraction?**

**? Can you make up another division by a whole number? (For example, twelve divided by three.)**

Support this division with a calculator if necessary, clarifying which are whole numbers.

**? When you divide by a whole number, is the answer larger or smaller than the number you started with?**

Record:

$$10 \div 2 = 5$$

$$12 \div 3 = 4$$

$$20 \div 4 = 5$$

When you divide a whole number by a whole number (two or more), the answer is smaller than the number you started with.

Let the child explore on a calculator what happens when you divide by one.

Look back at previous recordings, for example from Spotlight 1.

**? What do you think happens to a whole number when you divide it by a fraction (less than one), such as dividing by a half or a quarter?**

Support this by looking back at previous recordings, helping the child to identify calculations that are divisions by fractions (halves or quarters, etc.) and comment on what they notice.

$$10 \div \frac{1}{2} = 20 \text{ (How many halves in ten?)}$$

$$4 \div \frac{1}{4} = 16 \text{ (How many quarters in four?)}$$

$$12 \div \frac{1}{2} = 24$$

- ? When you divide by a number less than one, is the answer smaller or larger than the starting number? (Larger)**

If the child is uncertain, briefly review the previous activities, particularly the practical tasks.

Let the child illustrate and explain using practical equipment, showing that when you divide by a fraction the answer is larger than the starting number.

'Let's write two sentences to show what happens when you divide by a whole number, compared with dividing by a fraction. We can take what we write back to class to show the others.'

When you divide by a number less than one (a fraction) the answer is larger than the starting number.

But when you divide by a whole number (two or more) the answer is less than the starting number.

- ? If you worked out twelve divided by a quarter, will the answer be more or less than twelve? (More)**



- ? Using only the digits 1, 2, 3 and 4 (you can use these digits to make fractions) write two division calculations: one with an answer which is more than the number you chose to start with; and one with an answer which is less than the number you chose to start with.**

Display the two general statements from the previous page in class. 'Do you think these two statements are true? Are they always true?' (Give the children calculator experience of dividing by one and by zero.)

- ? What happens when you multiply a number by a fraction?**

## Spotlight 3

Interprets division as sharing but not as grouping (repeated subtraction) so is unable to interpret a calculation such as  $12 \div \frac{1}{2}$

### Opportunity for: developing mental images



### Draw it

Time 15–20 minutes

#### Resources

- Cuisenaire or other rods
- Paper fraction pieces or *Fractions* (Resource sheet 42)
- Apples and other items to divide, or dough
- Chocolate bars or pictures

#### Key vocabulary

- |            |                        |
|------------|------------------------|
| halves     | divided by             |
| quarters   | how many halves in...? |
| whole ones | divided into halves    |
| fractions  |                        |

### Teaching activity

Show the child the equipment.

‘We are going to work on dividing numbers by a half and by a quarter and I want you to draw what we do, or make a picture with anything on the table – or you can get anything else you need if you prefer.’

- ? At her party, Jane has allowed a quarter of a pizza for each person. How many quarters can she get from three pizzas?**
- ? Can you draw a picture of that?**
- ? Can you tell me what to write to make a correct division number sentence?**

$3 \div \frac{1}{4} = 12$   
She can make 12 quarters.

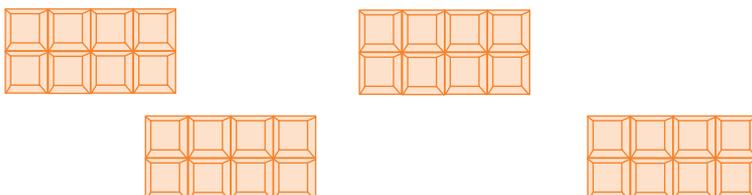
- ? Can you show three divided by a quarter using a different kind of picture, or with some of the equipment? (For example, choosing a rod to represent one whole one and using three of those whole ones and twelve of the related quarter rods.)**

If the child seems stuck, let them make models with the dough, or with paper and encourage them to try to ‘see’ calculations in several different ways.

- ? Can you choose something from the equipment, or draw a picture, to show six divided by a quarter or how many quarters are there in six whole ones?**

Tell me about your pictures.

Show the child some chocolate bars or pictures.



**? Can you make up a division calculation with these chocolate bars, dividing by a fraction?**

*(There are several ways to do this: dividing by halves, quarters, eighths.)*

If that is a bit too challenging, suggest which fraction they could divide by, for example halves, then give the child a while to think about it.

If they need further help, you could start a sentence for them. 'Four chocolate bars divided into halves...' (This could continue with something like 'how many children can have half a bar?')

**? If the answer to one of my dividing by a fraction calculations was sixteen, what could the question have been?** *(There are several possible questions, for example how many quarter apples can I make from four whole ones?)***? Can you draw a picture of what you had in your head when you worked that out?**

Depending on the child's drawings and the range of images they can use confidently, or the range of equipment they can work with, you might want to repeat this Spotlight before you move on.

**? What is your favourite way to work out dividing by a fraction?**

## Spotlight 4

Interprets division as sharing but not as grouping (repeated subtraction) so is unable to interpret a calculation such as  $12 \div \frac{1}{2}$

### Opportunity for: reasoning about numbers



### Calculator challenge

Time 15–20 minutes

#### Resources

- Number cards and symbol cards (Resource sheets 1, 2, 8 and 27)
- Previous recordings
- Calculator

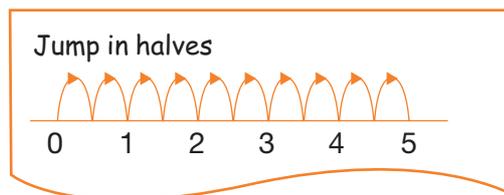
#### Key vocabulary

- |            |                        |
|------------|------------------------|
| halves     | how many halves in...? |
| whole ones | divided into halves    |
| fractions  | multiplied by          |
| divided by |                        |

### Teaching activity

‘We are going to do some more dividing by fractions, so that you will feel confident that you can work these out.’

Choose an earlier recording or draw a number line like this.



**? Tell me a division that this number line picture shows. (For example, ‘How many halves make five?’)**

Support the child by counting the hops. So that there are ten hops of one half, the number line is to five and each hop is a half.

Lay out the relevant cards.

**? Can you use some of these cards to show a division that matches the number line picture?**

10	$\frac{1}{2}$	5	÷	×	=
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**? Can you show this another way? ( $5 \div \frac{1}{2} = 10$ ,  $5 \div 0.5 = 10$ )**

Help the child to key in the first two numbers on a calculator but not to press the = key yet.

**? Does the answer get bigger or smaller when you divide it by a number less than one?**

**? Do you know which number is going to come on the screen when you press the = key?**

You might need to demonstrate and talk through what the calculator has done, linking it to the number line picture. If the child still finds this difficult, move on to a calculator challenge, giving the child plenty of time and examples (and plenty of opportunity to talk in order to make sense of what is happening).

'Now I'm going to give you some challenges with the calculator and you have to guess what the answer will be before you press the  key.'

'Key in six divided by nought point five.'

**? Can you work out which number will show when you press the equals key?**

Repeat with:  $8 \div 0.5$ ,  $10 \div 0.5$ ,  $12 \div 0.5$ .

**? Can you continue the pattern?**

Give plenty of further examples, continuing this in your next session if the child needs more help to identify the patterns and to work out the answers themselves.

Examples could be:

- multiples of four divided by a quarter, for example  $4 \div 0.25$ ;
- more even numbers divided by 0.5.

**? What do you think will happen if you divide odd numbers by a half? (The answer is double the starting number, just like even numbers.)**

You might want to record some of their calculations, but make sure you give them plenty of time when you are not looking at the display so that they can try things out for themselves.



**? Can you explain why this division and this multiplication have the same answer?**

**? Can you make another two calculations that link together in a similar way?**

$$4 \div 0.25 = 16$$

$$4 \times 4 = 16$$

## Spotlight 5: a learning check

Interprets division as sharing but not as grouping (repeated subtraction) so is unable to interpret a calculation such as  $12 \div \frac{1}{2}$

### Opportunity for: discussing and explaining

### Picture match game

Time 10–15 minutes

#### Resources

- *Picture match game board* (Resource sheet 43)
- *Picture match game cards* (Resource sheet 44)
- At least two players
- Calculators
- Rewards such as counters, stickers, etc.

#### Check: does the child use key vocabulary?

halves	how many halves in...?
quarters	divided into halves
whole ones	hops of a half
fractions	divided into half
divided by	half of

### Teaching activity

Put out the game board (Resource sheet 43) for **Picture match game** and give the children a chance to look at it carefully and to count the jumps, and so on. Encourage them to talk to each other about what they see.

Put out, face up, an even number of the picture match cards from Resource sheet 44. (If you wish, you can just use the cards in the left-hand column of the sheet. These all go with pictures 1 and 3. The cards in the right-hand column all go with the pictures, except for the last one, which doesn't match any of the pictures.)

#### How to play

1. The players take turns to pick a card, read it to the other player and decide which picture that number sentence can go with. (Some sentences will go with more than one picture. Some sentences might not go with any of the pictures.)
2. If everyone agrees that they are right, the player will win a reward. If they aren't right, the card then passes to the other player, and, if they are right, they get another turn.
3. Take turns until all the cards are used up and the player with the most rewards wins.

**?** How did you decide that card went with that picture?

**?** Will that number sentence go with any of the other pictures as well?

#### Variations

- Pairs can play cooperatively or play against another pair.
- Don't provide any cards but ask the players to make cards for all four pictures and then give the cards to another pair.
- Let the children make four pictures and related cards of their own, which they should give to another pair to sort out.

**Learning outcomes**

By the end of this set of activities, children should be able to:

- tackle related learning tasks with increased motivation and confidence;
- use and understand connected mathematical vocabulary;
- interpret a calculation in which there is dividing by a fraction;
- explain their drawings and mental images when dividing by fractions;
- solve real-life problems involving division by fractions;
- begin to generalise about number patterns when dividing by a number less than one.