

Has difficulty in choosing suitable methods for calculations that cross boundaries: subtraction

Opportunity for: making decisions

Resources:

- Empty number lines
- Place value (arrow) cards
- Base 10 equipment, straws, etc.
- Four bead strings
- Calculations on sticky notes

Key vocabulary

- | | |
|------------|------------------------------|
| count on | still the same number |
| count up | boundary |
| count back | next multiple of ten/hundred |
| subtract | take away |
| partition | leaves |
| equals | |

Teaching activity

Time 10–15 minutes

‘We’re going to do some work on subtraction, looking particularly at what you can do when you have to cross a boundary.’

? What would you do to subtract nineteen from three hundred and eighteen?

Let the child use any equipment they choose.

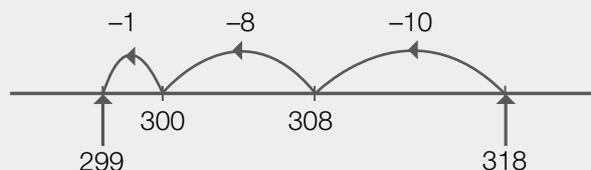
Listen and observe carefully, then follow on from what the child does.

If the child is having difficulty with this, ask them how they like to see big numbers – bead strings? number lines?

Count out three hundred and eighteen on four bead strings and ask the child to count back nineteen.

Support the child to see the relationships in the numbers: that nineteen is one more than eighteen so the answer is one less than three hundred.

Demonstrate counting back on a number line to check.



Write these calculations on sticky notes:

- | | |
|---------|---------|
| 62 – 31 | 74 – 29 |
| 62 – 35 | 86 – 33 |
| 74 – 21 | 86 – 37 |

? Look carefully at these calculations and sort them into ones that don’t cross boundaries and ones that do.

Support the child to put the calculations that cross boundaries in a group.

Do not cross boundaries	62 − 31	Do cross boundaries	62 − 35
	74 − 21		74 − 29
	86 − 33		86 − 37

Let the child work out a few of the calculations and explain which are easier and why.

If the child has difficulty, do some more of the easier ones, for example: $29 - 13$.

Let the child talk you through what they do.

Support by counting up on a number line (or counting back if the child chooses this method).

$29 - 13 = 16$

Establish that a method of column subtraction that seems easy to do for $62 - 31$ will not work for some of the calculations on the sticky notes, such as $62 - 35$.

Move on to some of the calculations that cross boundaries, starting, for example, with $60 - 35$.

? How would you work that out?

Listen and observe.

Demonstrate two methods, counting up on a number line and partitioning with place value cards, in column format.



? About how much is the answer likely to be?

? To help us work it out, make sixty-two and thirty-five with place value cards.

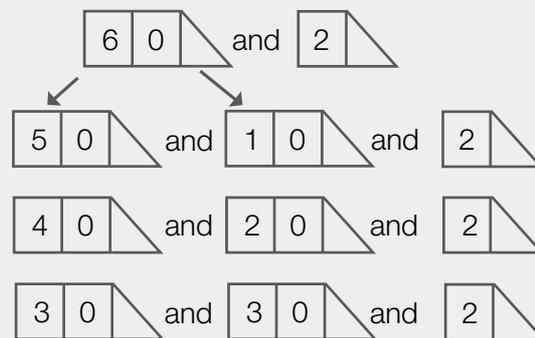
Ask the child how to partition both numbers.

Demonstrate how we can partition sixty-two again to make it easier to subtract when we have to cross a boundary. Show that sixty-two has been partitioned into fifty and ten and two.

$$\begin{array}{r} 62 \\ - 35 \\ \hline \end{array}$$

It is crucial that the child is absolutely sure about this. If necessary, stop at this point and show many ways that sixty-two can be partitioned – but each time it is still sixty-two. It is just broken down in different ways, just like we can break down ten into four and six, two and eight, and so on.

62 partitioned



Refer to 2 Y4 +/- for further work on partitioning.

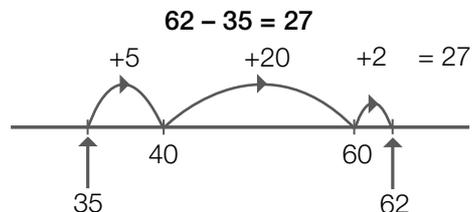
Work through the calculation with the child moving the place value cards, asking the child to tell you what they think is going on.

? Is the answer close to your estimate?

? How can we check we are right?

Listen and observe and try to build on what the child says.

Counting up on the number line is one efficient way to check. The diagram shows counting up; if you have seen the child choose counting back as a preferred way of subtracting on a number line, follow their lead.



Repeat with other examples, each time establishing that it is the partitioning then partitioning the number again that is helping us to calculate when we cross boundaries.

? What shall I write down to help us remember?

Spotlight 1

Has difficulty in choosing suitable methods for calculations that cross boundaries: subtraction

Opportunity for: identifying and using number relationships

.....

Counting up

Time 15–20 minutes

Resources

- Sticky notes with calculations
- Place value (arrow) cards
- Bead strings
- Empty number lines

Key vocabulary

count on	still the same number
count up	boundary
count back	next multiple of ten/hundred
subtract	take away
partition	leaves
equals	

Teaching activity

‘Today we are going to work on some calculations that all have something that is the same about them. I wonder if you will be able to work out what that is.’

Display the calculations on sticky notes.

100 – 96 400 – 399 500 – 496 1004 – 996

Ask the child to choose one to work out, for example 100 – 96.

? What would you like to use to help you?

Lead on from what the child suggests, but get them to reflect on their calculation and consider its efficiency if appropriate.

? Make an estimate first. About how big is the answer?

Then support the child counting up on fingers and show how that can be done on a number line.

? Look carefully at the rest of the calculations. Which other ones would you choose to do in a similar way?

All of them could be done with a counting on method.

? Do you think we would get the same answer if we counted back from one hundred to ninety-six?

Work through a few more examples.

? How can you check you have the right answer?

Encourage the child to see that, if you add your answer back on, you should get back to the number you started with.

$$1004 - 996 = 8 \quad \rightarrow \quad 996 + 8 = 1004$$

? What do you think makes all of these similar? (They can all be worked out in a similar way. There is a small gap between the two numbers and they all cross either a hundred or a thousand boundary.)

? Can you write a subtraction that has a large difference between the two numbers?



‘Write three subtraction calculations that cross the thousand boundary.’

Spotlight 2

Has difficulty in choosing suitable methods for calculations that cross boundaries: subtraction

Opportunity for: comparing methods

Quicker or slower?

Time 15–20 minutes

Resources

- Empty number lines
- Place value (arrow) cards
- Bead strings
- Bundles of straws or other Base 10 equipment

Key vocabulary

- | | |
|------------|------------------------------|
| count on | still the same number |
| count up | boundary |
| count back | next multiple of ten/hundred |
| subtract | take away |
| partition | leaves |
| equals | |

Teaching activity

‘We are going to look at some subtraction calculations today, check which ones are reasonably easy. Then we are going to work on the harder ones with number lines and place value cards.’

First look at some calculations that are simple to do by subtraction in columns, for example:

384 – 122 and 368 – 245



? What makes these simpler to do?

? Make an estimate for that one. (Note: It is important to ensure that the value of the digits is articulated. For example, ‘eighty subtract twenty’, not ‘eight subtract two’.)

Then look at some calculations crossing boundaries (note how these relate to the previous ones having the same starting number).

384 – 155 and 368 – 249

? What makes these different?

? What would you do to solve these?



Observe and listen. Then, if necessary, help the child to reflect on their process. Build on the child’s chosen method, supporting with place value cards or number lines as appropriate.

See the following examples.

$$\begin{array}{r} 384 \\ - 155 \\ \hline \end{array}$$

$$\begin{array}{ccc} 300 & 80 & 4 \\ & \swarrow & \searrow \\ & 70 & 10 \end{array}$$

300	70	14	
100	50	5	
200	20	9	
$\swarrow \quad \downarrow \quad \swarrow$ 229			

Counting up

$+5 \quad +40 \quad +100 \quad +80 \quad +4 = 229$

5	/
---	---

4	0	/
---	---	---

1	0	0	/
---	---	---	---

8	0	/
---	---	---

4	/
---	---

Stop and ask the child questions as you are working through it.

If the child has chosen partitioning as their preferred method, continue with text as follows. If not, give another three-digit example for them to work through, such as $264 - 148$. Finish with the last question of this Spotlight.

? Why are we doing this partitioning?

Particularly identify the two-stage partitioning of the eighty so that there are enough units (14) to be able to take away the five units.

Make absolutely sure that the child accepts that you can partition eighty-four into eighty and four, but you can go on partitioning it until the numbers are easier to do the subtraction of the units.

Make sure the child sees that the number partitioned is still eighty-four! And that doing the partitioning is not subtracting!

If you are not sure the child grasps this, do some more partitioning of numbers, asking at each stage:

? Are we still partitioning, or have we started subtracting yet?

8	4	/
---	---	---

8	0	/
---	---	---

4	/
---	---

7	0	/
---	---	---

1	0	/
---	---	---

4	/
---	---

still 84

6	0	/
---	---	---

2	0	/
---	---	---

4	/
---	---

still 84

5	0	/
---	---	---

3	0	/
---	---	---

4	/
---	---

still 84

Repeat with another pair of three-digit numbers, for example $264 - 148$.

? Which number do we need to partition and partition again? Why?

? Which method do you prefer to help you subtract across boundaries?

Spotlight 3

Has difficulty in choosing suitable methods for calculations that cross boundaries: subtraction

Opportunity for: reasoning about numbers

.....

Subtraction boundaries

Time 15–20 minutes

Resources

- Four bead strings
- Place value (arrow) cards
- Empty number lines

Key vocabulary

count on	still the same number
count up	boundary
count back	next multiple of ten/hundred
subtract	take away
partition	leaves
equals	

Teaching activity

‘We’re going to do some more work on boundaries today, first where we cross just the tens boundary, then where we cross the hundreds boundary.’

? What is three hundred and eighteen subtract seven?

? What about three hundred and eighteen subtract eight?

? What about three hundred and eighteen subtract nine? How could you work that out?

If the child needs more support to visualise what is going on, you could use four bead strings to show three hundred and eighteen.

Clarify for the child that in that last calculation you had to cross the tens boundary.

If the child is unclear about that, you might be better to work through another example with crossing just the tens barrier rather than moving on.

Focus on estimate, calculate and check and on number line calculations until the child is accurate.



‘Now we are going to work through an example where you have to cross the hundreds boundary.’

? Three hundred and eighteen subtract one hundred and seventy-one?

? Make an estimate first.



Ask the child to show their way of calculating. It is vital to get the child talking about their reasoning as they calculate and using their recording to reflect their reasoning.

Three alternatives are illustrated below. Counting up on a number line might be the more successful method for children who have had difficulty previously.

Having established a preferred method for them that enables them to be reasonably accurate and efficient, choose a further calculation for them to practise, for example $454 - 173$.

Counting up

$327 - 171$

$171 \quad 180 \quad 200 \quad 300 \quad 320 \quad 327$

$\begin{array}{|c|} \hline 9 \\ \hline \end{array} \begin{array}{|c|c|} \hline 2 & 0 \\ \hline \end{array} \begin{array}{|c|c|c|} \hline 1 & 0 & 0 \\ \hline \end{array} \begin{array}{|c|c|} \hline 2 & 0 \\ \hline \end{array} \begin{array}{|c|} \hline 7 \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline 1 & 5 & 6 \\ \hline \end{array}$

Work through the example on a number line, showing how you can go up to the nearest multiple of ten first, then you can choose a range of different ways to count on; in this example, going to the nearest multiple of a hundred next. Be guided by the child's suggestions of suitable hops to make.

Counting back

$327 - 171$

$171 \quad 180 \quad 200 \quad 300 \quad 327$

$\begin{array}{|c|} \hline 9 \\ \hline \end{array} \begin{array}{|c|c|} \hline 2 & 0 \\ \hline \end{array} \begin{array}{|c|c|c|} \hline 1 & 0 & 0 \\ \hline \end{array} \begin{array}{|c|c|} \hline 2 & 7 \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline 1 & 5 & 6 \\ \hline \end{array}$

Using partitioning

$327 - 171$

$\begin{array}{|c|c|c|} \hline 3 & 0 & 0 \\ \hline \end{array} \begin{array}{|c|c|} \hline 2 & 0 \\ \hline \end{array} \begin{array}{|c|} \hline 7 \\ \hline \end{array}$
 $\begin{array}{|c|c|c|} \hline 2 & 0 & 0 \\ \hline \end{array} \begin{array}{|c|c|c|} \hline 1 & 0 & 0 \\ \hline \end{array} \begin{array}{|c|c|} \hline 2 & 0 \\ \hline \end{array} \begin{array}{|c|} \hline 7 \\ \hline \end{array}$
 subtract $\begin{array}{|c|c|c|} \hline 1 & 0 & 0 \\ \hline \end{array} \begin{array}{|c|c|} \hline 7 & 0 \\ \hline \end{array} \begin{array}{|c|} \hline 1 \\ \hline \end{array}$

 $\begin{array}{|c|c|c|} \hline 1 & 0 & 0 \\ \hline \end{array} \begin{array}{|c|c|} \hline 5 & 0 \\ \hline \end{array} \begin{array}{|c|} \hline 6 \\ \hline \end{array}$

When the child can work with place value cards, partitioning and partitioning again in just the hundreds, they could move on to calculations where you need to partition in the hundreds in two stages as well as in the tens, for example:

? When we come to the end of a calculation, what do we need to remember to do? (Remember to check.)



'Write a calculation that starts with three hundred and something in which you have to cross both the tens boundary and the hundreds boundary.'

Spotlight 4

Has difficulty in choosing suitable methods for calculations that cross boundaries: subtraction

Opportunity for: exploring larger numbers

Subtracting from 1000

Time 20–25 minutes

Resources

- Money
- String and measuring tape
- Base 10 equipment such as bundles of straws

Key vocabulary

- | | |
|------------|------------------------------|
| count on | still the same number |
| count up | boundary |
| count back | next multiple of ten/hundred |
| subtract | take away |
| partition | leaves |
| equals | |

Teaching activity

? Can you remember what we did last time?

‘Today we are going to work on subtracting from a thousand so you will be able to play a game where you have to be able to do that.’

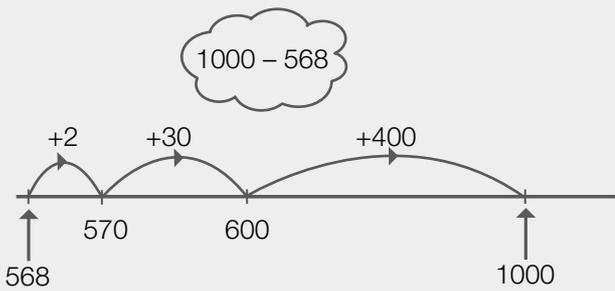
? How would you work out the answer to this? 1000 – 568

 Remind the child to estimate first. Take time with this and establish that the answer must be four hundred and something.

Observe and listen as the child works it out.

This isn’t a good example for using place value cards and partitioning! Encourage a mental method, with jottings.

If the child has difficulty with this, show a number line method.



4 0 0 / 3 0 / 2 = 4 3 2 /

‘We can jump two to 570 first because that is the nearest multiple of ten, then we can jump 30 to get to 600.’

? How big is the jump from 600 to 1000?



Show a bundle of a thousand straws or a Base 10 thousand block.

? How could you subtract ten from a thousand?

? What would your answer be, roughly?

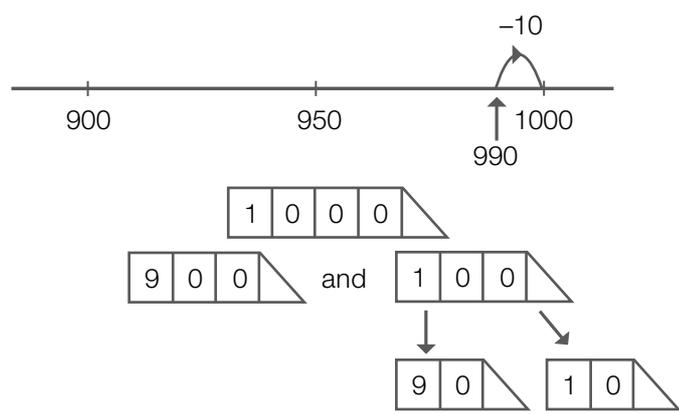
Establish that it would be nine hundred and something, close to a thousand. (*The actual answer is 990.*)

Demonstrate how if you add ten back on again you get back to a thousand.

Now model this subtraction of ten from a thousand in different ways.

£10 is 1000 pence $£10 - 10p = £9.90$

1000 cm of string (10 m) – 10 cm = 9 m 90 cm



? Look carefully at the answers and tell me what looks the same.

Ask the child to comment on partitioning as a method for this calculation.

Demonstrate how you could work out $1000 - 10$ by partitioning because you can partition a thousand.

? What do you think is the best way of calculating $1000 - 10$?



? Subtract 999 000 from 1 000 000. ($1\ 000\ 000 - 999\ 000 = 1000$)

? What would be the most efficient method? No calculator available!

Spotlight 5: a learning check

Has difficulty in choosing suitable methods for calculations that cross boundaries: subtraction

Opportunity for: explaining and discussing

Frog in the well

Time 20–25 minutes

Resources

- Blank spinners (Resource sheet 13)
- Pencil and paper clip to work the spinner
- Climbing up (Resource sheet 28)
- At least two children
- Empty number line for each child or pair
- Small counter for each team
- Timer

Check: does the child use key vocabulary?

- | | |
|------------|------------------------------|
| count on | still the same number |
| count up | boundary |
| count back | next multiple of ten/hundred |
| subtract | take away |
| partition | leaves |
| equals | |

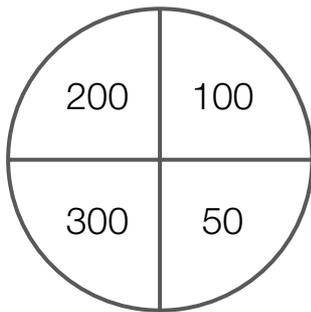
Teaching activity

‘This game, **Frog in the well**, will help you with adding and subtracting with larger numbers. Today we are going to work with a thousand.’

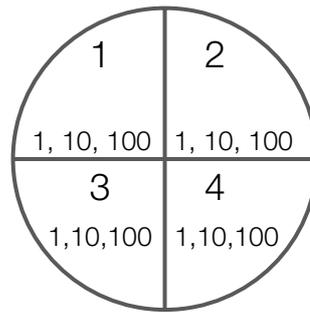
Set the game in some kind of context, a climber going up a 1000 metre high cliff or a frog climbing out of a 1000 metre deep well.

Children can cooperate to play this game in pairs or a larger team, or they can compete against each other.

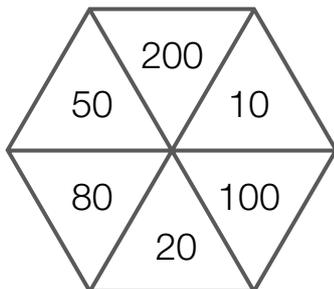
The spinner works by trapping the paper clip in the centre of the spinner with the pencil, then flicking round the paper clip with a finger of the other hand. If the paper clip stops on a line, the player chooses which of the two sections they want.



spinner A



spinner B



spinner C

Decide which spinner to play with and the height of the well or cliff.

Each player needs to draw their well or cliff first before the start of the game or use *Climbing up* (Resource sheet 28).

Decide whether you are going to play from a thousand down to zero or up from zero. This example starts at zero and uses the easiest spinner A.

You could start the timer once the players know what to do.

How to play

1. Each team, pair or single player puts their counter on zero on their vertical number line or on Resource sheet 28 and takes turns to spin the spinner. They should record their numbers.
2. If they spin, for example one hundred, they move their counter from zero to 100 and write down the number of metres they still have to climb, in this case, 900 metres.
3. If everyone agrees they are right, the next player takes a turn with the spinner.
4. If players don't agree, the player who has moved has one more attempt to do their calculation correctly. If they are still wrong, they go back to where they were so they miss that turn.
5. They must try to get to the top in less than 15 minutes, then next time try to beat their personal best.
6. The winning team, if they are competing, gets to the top or beyond the top first, but they can only win if they can add up the total of all of their jumps! It should come to a thousand or more.

Variations

- ↑ ● Play with a different spinner, for example spinner B. On this spinner, if you spin, for example, three you can choose whether you want to move three ones or three tens or three hundreds.
- ↑ ● If you play with spinner B you could change the rules so that if you spin, for example, three, you can choose any combination of the moves, as long as you move a total of three. So you could move one 100, one 10 and one 1.
- ↑ ● Spinner C is a bit harder. You will need to keep your recordings very clear.
- ↑ ● Play with any of the spinners and start from a thousand. Still say at each turn how far you have to go.
- ↑ ● Make the well or cliff a bit higher, for example 1050.

Learning outcomes

By the end of this set of activities children should be able to:

- tackle related learning tasks with increased motivation and confidence;
- use and understand connected mathematical vocabulary;
- choose suitable methods of calculating when subtraction calculations cross boundaries;
- understand partitioning with place value cards when one or more boundary is crossed;
- estimate, calculate and check to see if their work is right.